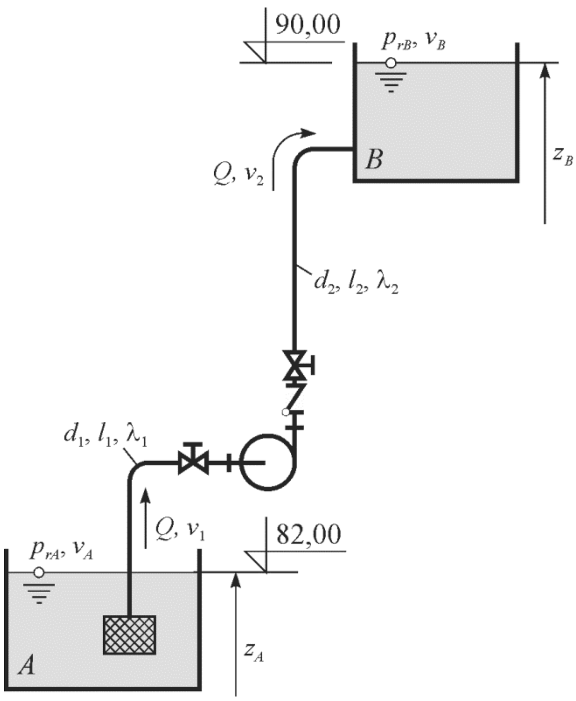
Exercise 1: Centrifugal pump, whose performance curves at n = 1450 rpm are given in the Table 1, pumps the water from the reservoir A to the reservoir B. Reservoir elevations are given in the Figure.



Suction and discharge pipeline characteristics are:

l_1 = 10 m, d_1 = 100 mm, λ_1 = 0,025, $\Sigma\zeta_1$ = 2;

l_2 = 95 m, d_2 = 80 mm, λ_2 = 0,027, $\Sigma\zeta_2$ = 12;

Problems:

1. Find the pump flow rate, head and shaft power if the pump rotation speed is at n = 1450 rpm.
2. Find the pump rotation speed at which the flow will increase for 25 % (compared to the flow found above) and equivalent shaft power then.
3. Find the pump flow rate, head and shaft power if the pump rotation speed is at n = 1300 rpm.

Table 1: Pump performance characteristics at n = 1450 rpm

Q [L/s]	0	2	4	6	8	10	12	14
Y [J/kg]	147	149	149	146	137	122	100	76
η [%]	0	40	63	75	75	70	58	42

Solution (1) – pump operating in a simple system

Problem is solved using Bernoulli equation for points on surfaces of water in reservoirs A and B:

$$\frac{p_a + p_{rA}}{\rho} + \frac{v_A^2}{2} + gz_A + Y_p = \frac{p_a + p_{rB}}{\rho} + \frac{v_B^2}{2} + gz_B + \sum \Delta y_{gs} + \sum \Delta y_{gp} \qquad (1)$$

p_{rA} and p_{rB} are relative pressures on surfaces of the water in reservoirs A and B ($p_r = +p_m$ if it is overpressure, $p_r = -p_v$ if it is underpressure). Ofcourse, if they are indeed water reservoirs, they are in most cases open to atmospheric pressure so relative pressure is $p_r = 0$. In every other case relative pressure must be taken into account.

Terms v_A and v_B in equation (1) represent velocities in reservoirs if that velocities are not negligently small (e.g. if "reservoir" is actually water course), z_A and z_B are heights of water levels in reservoirs. Y_p represents head and Δy_{gs}, Δy_{gp} loss of specific flow energy (hydraulic loss) in suction (s) and discharge (p) pipeline. Atmospheric pressure (p_a) is found on the both sides of equation (1) and therefore can be eliminated, so that means Bernoulli equation is also applicable with relative pressures, and this is the form of it:

$$\frac{p_{rA}}{\rho}+\frac{v_A^2}{2}+gz_A+Y_p=\frac{p_{rB}}{\rho}+\frac{v_B^2}{2}+gz_B+\sum\Delta y_{gs}+\sum\Delta y_{gp} \quad (2)$$

Hydraulic losses on suction and discharge pipelines can be separated on friction losses and local losses so equation (2) has this form now:

$$\frac{p_{rA}}{\rho}+\frac{v_A^2}{2}+gz_A+Y_p=\frac{p_{rB}}{\rho}+\frac{v_B^2}{2}+gz_B+\frac{v_1^2}{2}\left(\lambda_1\frac{l_1}{d_1}+\sum\zeta_1\right)+\frac{v_2^2}{2}\left(\lambda_2\frac{l_2}{d_2}+\sum\zeta_2\right) \quad (3)$$

In hydraulic problems, in most cases with Bernoulli equation, continuity equation is also used to give link between velocities in suction and discharge pipelines::

$$v_1A_1=v_2A_2 \quad \rightarrow \quad v_1\frac{\pi d_1^2}{4}=v_2\frac{\pi d_2^2}{4} \quad (4)$$

Equations (3) and (4), with pump performance characteristics $Y_p(Q_p)$, given in Table 1, make a system of 3 non-linear algebraic equations with unknown head Y_p and velocities v_1 и v_2. However, in pump systems velocities are not the primary unknown, the flow is, so it is of intrest to make a system of equations where the flow is explicitly figuring as the only unknown. In that case continuity equation (4) should be used as a link between velocity and flow in pipelines:

$$Q=v_1\frac{\pi d_1^2}{4}=v_2\frac{\pi d_2^2}{4} \quad \rightarrow \quad v_1=\frac{4Q}{\pi d_1^2}, \quad v_2=\frac{4Q}{\pi d_2^2}$$

If the given links are used (terms referring to velocities in reservoirs stay unchanged because they are not velocities in pipelines) equation (4) becomes:

$$\frac{p_{rA}}{\rho}+\frac{v_A^2}{2}+gz_A+Y_p=\frac{p_{rB}}{\rho}+\frac{v_B^2}{2}+gz_B+\frac{8Q^2}{\pi^2 d_1^4}\left(\lambda_1\frac{l_1}{d_1}+\sum\zeta_1\right)+\frac{8Q^2}{\pi^2 d_2^4}\left(\lambda_2\frac{l_2}{d_2}+\sum\zeta_2\right)$$

(5)

If equation (5) is rearranged so on the left side is only head of the pump:

$$Y_p=\frac{p_{rB}-p_{rA}}{\rho}+\frac{v_B^2-v_A^2}{2}+g(z_B-z_A)+\frac{8Q^2}{\pi^2 d_1^4}\left(\lambda_1\frac{l_1}{d_1}+\sum\zeta_1\right)+\frac{8Q^2}{\pi^2 d_2^4}\left(\lambda_2\frac{l_2}{d_2}+\sum\zeta_2\right) \quad (6)$$

Equation (6) and Table performances of the pump $Y_p(Q_p)$ make a system of 2 non-linear algebraic equations consisting of unknown flow and head of the pump. There are two ways of solving this system, numerically or graphically. The graphic solution will be shown below.

Expression on the right side of equation (6) is usually called “system characteristic”:

$$Y_c(Q)=\frac{p_{rB}-p_{rA}}{\rho}+\frac{v_B^2-v_A^2}{2}+g(z_B-z_A)+\frac{8Q^2}{\pi d_1^4}\left(\lambda_1\frac{l_1}{d_1}+\sum\zeta_1\right)+\frac{8Q^2}{\pi d_2^4}\left(\lambda_2\frac{l_2}{d_2}+\sum\zeta_2\right)$$

So it is to be said that pump system in stable regime works with flow Q with whom head of the pump is equal to the head demanded by the system:

$$Y_p(Q)=Y_c(Q)$$

In this problem, both of the reservoirs are open to atmospheric pressur so $p_{rA} = p_{rB} = 0$, and velocities in reservoirs are in most cases negligible ($v_A \approx 0$, $v_B \approx 0$). So equation (6) is now in more simple form, which is usually more commonly found in practice:

$$Y_p(Q) = g(z_B - z_A) + \frac{8Q^2}{\pi^2 d_1^4}\left(\lambda_1 \frac{l_1}{d_1} + \sum \zeta_1\right) + \frac{8Q^2}{\pi^2 d_2^4}\left(\lambda_2 \frac{l_2}{d_2} + \sum \zeta_2\right) = Y_c(Q)$$

(7)

Term $g(z_B - z_A) = const$ in system characteristic $Y_c(Q)$ represents useful part of head. Height difference between reservoirs ($z_B - z_A$) is in practice usually called “lifting height“ H_{geo}.

Other two terms in system characteristics are written as $\Delta y_g = bQ^2$ and ,they represent hydraulic losses in pipeline, are also known as dissipative part of head.

By putting known values, given in description of the problem, z_A, z_B, d_1, l_1, λ_1, $\Sigma\zeta_1$, d_2, …in equation (7) it is obtained:

$$Y_p(Q) = 78{,}453 + 0{,}9084Q^2 = Y_c(Q)$$

Where coefficient next to Q^2 is calculated in that way the flow is written in L/s and not m^3/s!

Working performance characteristics of the pump $Y_p(Q_p)$, $\eta_p(Q_p)$ and system characteristics $Y_c(Q)$ are shown graphically bellow. Using equation (7), in inter-section of pump performance characteristics and system characteristics a solution is obtained as operating system point RT_1 (of pump and pipeline).

From operating system point RT_1, flow, head and efficiency coefficient of the pump will be read (graphically), based on which required shaft power will be calculated:

$RT_1 \rightarrow$ $\quad Y_{p1} = 136{,}9\,\frac{\text{J}}{\text{kg}}$ $\quad Q_{p1} = Q_1 = 8{,}0\,\frac{\text{L}}{\text{s}}$ $\quad \eta_{p1} = 75{,}0\ \%$

$$P_{p1} = \frac{\rho Q_{p1} Y_{p1}}{\eta_{p1}} = 1{,}46\ \text{kW}$$

Solution (2) – regulating the pump to specified flow by changing the speed of rotation

Operating system point RT_2 is obtained on system characteristic $Y_c(Q)$ from requirement that flow Q_2 is 25 % greater than Q_1:

$$Q_2 = 1{,}25 Q_1 = 1{,}25 \cdot 8\ \text{L/s} = 10\ \text{L/s} \quad \rightarrow \quad RT_2$$

New performance characteristic of the pump $Y_p^*(Q_p)$ must go through operating system point RT_2 at unknown rotation speed n_2, pump must achieve the following operating parameters :

$RT_2 \rightarrow$ $\quad Y_{p2} = 169{,}7\,\frac{\text{J}}{\text{kg}}$ $\quad Q_{p2} = Q_2 = 10\ \text{L/s}$

To define rotation speed n_2 at which pump can achieve required operating parameters, it is required that on “original” performance characteristic of the pump $Y_p(Q_p)$ at n = 1450 rpm is found so-called “similar point“ $ST(Q_s, Y_s)$, point that with the change of rotation speed of pump $n \rightarrow n_2$ is being mapped to operating system point $RT_2(Q_{p2}, Y_{p2})$. From affinity laws it is know that by changing rotation speed, all points on performance characteristics of the pump are being mapped by appropriate “similar parables” which are all going through coordinate start (Q = 0, Y = 0). Another words, if through operational point of the pump $RT_2(Q_{p2}, Y_{p2})$ is found similar parable with this form:

$$Y_s(Q_s) = \left(\frac{Y_{p2}}{Q_{p2}^2} \right) Q_s^2 = 1{,}6891 Q_s^2$$

in cross section of this parable and performance characteristics of the pump $Y_p(Q_p)$ at n=1450rpm, is found similar point ST_2 of operational point $RT_2(Q_{p2}, Y_{p2})$. From similar point we can read:

$ST_2 \rightarrow \qquad Q_{s2} = 8{,}8 \frac{\text{L}}{\text{s}} \qquad$ (head$\rightarrow \qquad Y_{s2} = 131{,}6 \frac{\text{J}}{\text{kg}}$)

Finally, rotational speed of the pump n_2 , using affinity laws, is:

$Q_p/n = const \rightarrow \qquad n_2 = \frac{Q_{p2}}{Q_{s2}} n = \frac{10 \text{ L/s}}{8{,}8 \text{ L/s}} 1450 \, \text{min}^{-1} = 1647 \text{ min}^{-1}$

or $\qquad Y_p/n^2 = const \rightarrow \qquad n_2 = \left(\frac{Y_{p2}}{Y_{s2}} \right)^{\frac{1}{2}} n = \left(\frac{169{,}7 \text{ J/kg}}{131{,}6 \text{ J/kg}} \right)^{\frac{1}{2}} 1450 \, \text{min}^{-1} = 1647 \text{ min}^{-1}$

To find required shaft power of the pump in operational point RT_2, except Q_{p2}, Y_{p2}, efficiency coefficient η_{p2} is also needed. Operational point RT_2 is found on performance characteristic of pump $Y_p^*(Q_p)$ at n_2 = 1647 rpm, so efficiency coefficient η_{p2} can’t be read because characteristic $\eta_p(Q_p)$ is found at n = 1450 rpm, $\eta_{p2} \neq \eta_p(Q_{p2})$!

This problem is solved using affinity laws to make a new characteristic $\eta_p^*(Q_p)$ at n_2 = 1647 rpm, so now at this characteristic it can be read $\eta_{p2} = \eta_p^*(Q_{p2})$. However it is a lot easier to use the fact that at “similar parable” $Q_p/n = const$ and $Y_p/n^2 = const$, is also $\eta_p = const$, so all points on “similar parable” have equal values of efficiency coefficients! So efficiency coefficient η_{p2} in operational point $RT_2(Q_{p2}, Y_{p2})$ is equal to efficiency coefficient in similar point $ST_2(Q_{s2}, Y_{s2})$ which can be read from characteristic $\eta_p(Q_p)$ at n = 1450 rpm:

$ST_2 \rightarrow \qquad Q_{s2} = 8{,}8 \frac{\text{L}}{\text{s}} \qquad \eta_{p2} = \eta_{s2} = 73{,}6\ \%$

Finally, required shaft power:

$$P_{p2} = \frac{\rho Q_{p2} Y_{p2}}{\eta_{p2}} = 2{,}3 \text{ kW}$$

In practice it is rare to find pump systems in which rotational speed is deliberately increased over nominal speed of operating electromotor. With increased rotational speed, required shaft power is also increased, so the operational electromotor should be in advance dimensioned for increase in required power.

Solution(3) – regulating the pump to specified flow by changing the speed of rotation

Operational point of the pump RT_3 is found on inter-section of $Y_c(Q)$ and performance characteristic $Y_p^{**}(Q_p)$ at n_3 = 1300 rpm. Performance characteristic of the pump $Y_p^{**}(Q_p)$ is acquired by recalculating characteristic $Y_p(Q_p)$ at n = 1450 rpm using affinity laws:

$$Q_p^{**} = \frac{n_3}{n} Q_p \qquad Y_p^{**} = \left(\frac{n_3}{n}\right)^2 Y_p \qquad \eta_p^{**} = \eta_p$$

Recalculating results are shown in Table.

Performance characteristics of the pump at n = 1450 rpm:

Q_p [L/s]	0	2	4	6	8	10	12	14
Y_p [J/kg]	147	149	149	146	137	122	100	76
η_p [%]	0	40	63	75	75	70	58	42

$\times\frac{1300}{1450}$ $\quad \times\left(\frac{1300}{1450}\right)^2$ $\quad =$

Performance characteristics of the pump at n_3 = 1300 rpm:

Q_p^{**} [L/s]	0	1,8	3,6	5,4	7,2	9,0	10,8	12,6
Y_p^{**} [J/kg]	118,2	119,8	119,8	117,4	110,1	98,1	81,2	61,1
η_p^{**} [%]	0	40	63	75	75	70	58	42

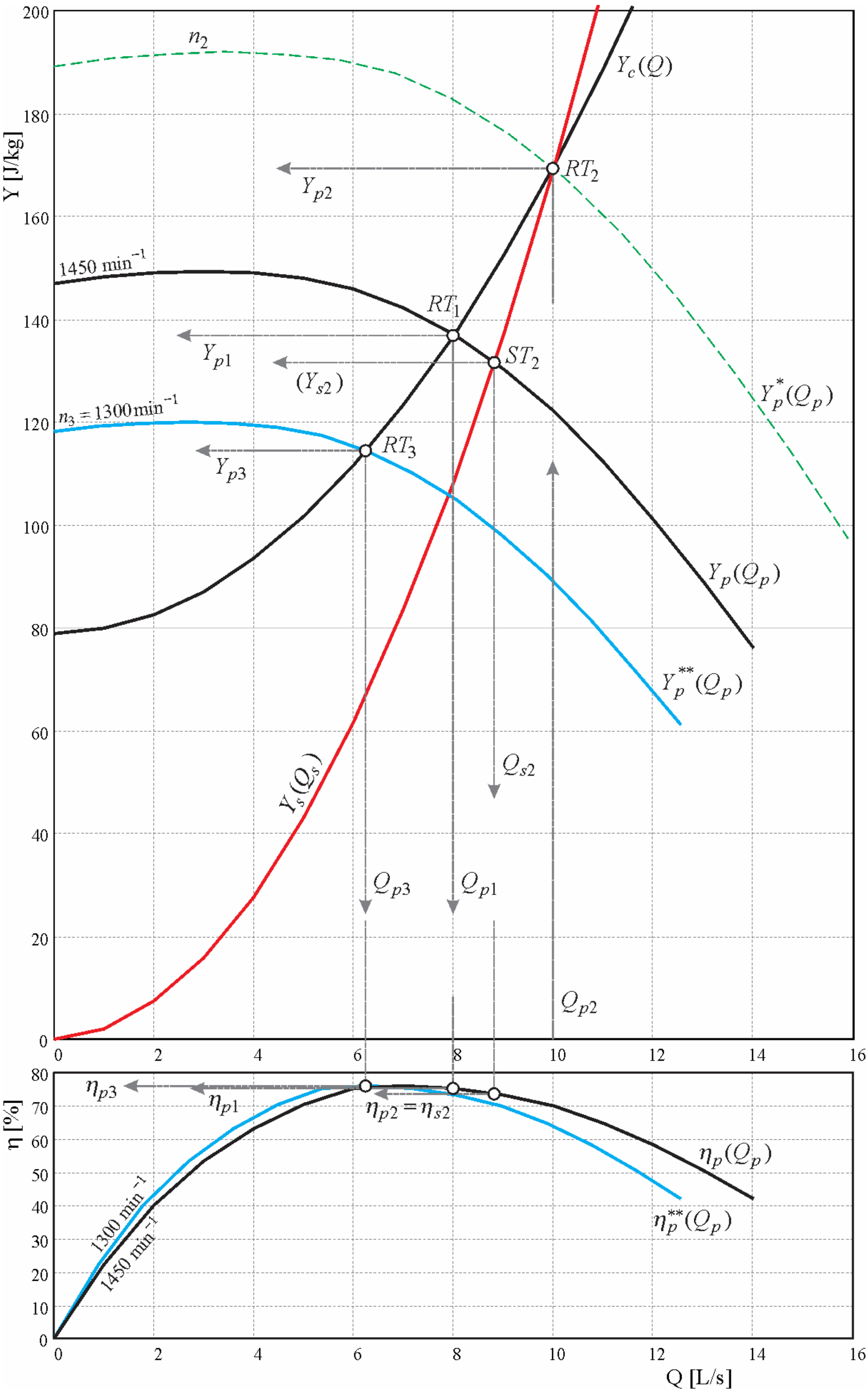
From operational point RT_3 flow, head, and efficiency coefficient can be read, which are used to find required shaft power:

$$RT_3 \rightarrow \qquad Y_{p3} = 114{,}3\frac{\mathrm{J}}{\mathrm{kg}} \qquad Q_{p3} = Q_3 = 6{,}3\frac{\mathrm{L}}{\mathrm{s}} \qquad \eta_{p3} = 76{,}1\ \%$$

$$P_{p3} = \frac{\rho Q_{p3} Y_{p3}}{\eta_{p3}} = 0{,}95\ \mathrm{kW}$$

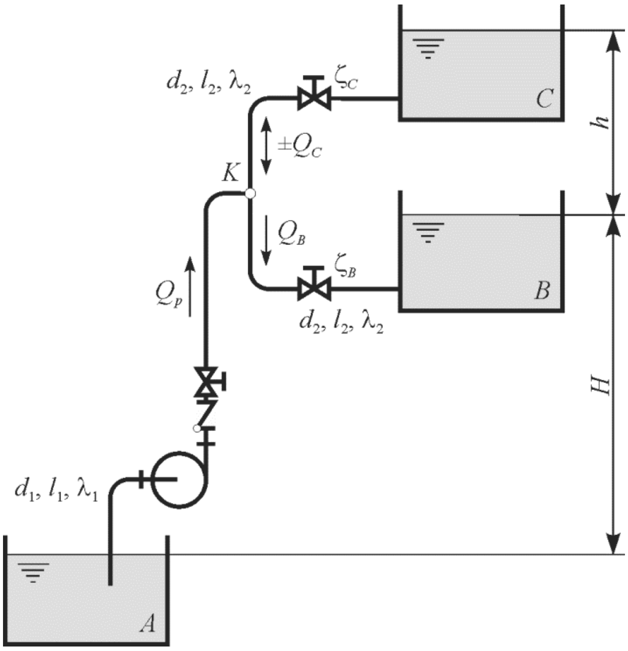
Homework:

1. Draw efficiency characteristic of the pump at n_2 = 1647 rpm which is acquired in solution (2) and find efficiency coefficient $\eta_{p2} = \eta_p^{*}(Q_{p2})$ at operational point RT_2. Compare with solution (2).
2. Find flow, head and required shaft power at 1450 rpm if reservoir B is at underpressure p_{vB} = 0,2 bar.
3. Find rotational speed of the pump at which pump is working with max efficiency coefficient. Both reservoirs are open. What is the shaft power required then?



Y [J/kg]
η [%]
Q [L/s]
n_2
$Y_c(Q)$
RT_2
Y_{p2}
1450 min^{-1}
RT_1
Y_{p1}
ST_2
(Y_{s2})
$Y_p^*(Q_p)$
$n_3 = 1300\,\text{min}^{-1}$
RT_3
Y_{p3}
$Y_p(Q_p)$
$Y_p^{**}(Q_p)$
$Y_s(Q_s)$
Q_{s2}
Q_{p3}
Q_{p1}
Q_{p2}
η_{p3}
η_{p1}
$\eta_{p2} = \eta_{s2}$
$\eta_p(Q_p)$
$\eta_p^{**}(Q_p)$
1300 min^{-1}
1450 min^{-1}

Exercise 2: Centrifugal pump whose performance curves at n = 1450 min^{-1} are given in the Table 1, pumps the water from reservoir A to reservoirs B and C, as shown in the Figure. The height differences between water levels in the reservoirs are: h = 10 m and H = 20 m. The pipe system consists of main line AK (d_1 = 100 mm, l_1 = 10 m, λ_1 = 0,022, $\Sigma\zeta_1$ = 10), and two equal branches KB and KC (d_2 = 60 mm, l_2 = 10 m и λ_2 = 0,023). Both branches are equipped with control valves with loss coefficients $\zeta_B = \zeta_C$ = 4, when the valves are fully open.



Problems:

1. Find inflows to both upper reservoirs B,C and shaft power of the pump if both control valves are fully open.
2. What should be the loss coefficient ζ_B, of the control valve in the branch KB, to ensure equal flow rates in both branches?
3. Find the pump rotational speed at which the inflow to the reservoir C becomes null (stops). Consider that both control valves are fully open.

Table 1: Pump performance characteristics at n = 1450 rpm

Q [L/s]	0	4	8	12	16	20	24	28	32
Y [J/kg]	510	530	535	530	510	481	432	373	294
η [%]	0	30	50	63	71	75	75	70	58

Solution (1):

Problem is solved using Bernoulli and continuity equation. Graphicall solution of Bernoulli equations is possible if Bernoulli equations are written for every system branch, so that every equation has only one unknown and that is flow.

It will be assumed, that at the inlets in reservoirs there are no irreversible valves, so it is possible under certain conditions that reservoir C can empty in reservoir B, another words, flow Q_C can have positive (to reservoir C) and negative (from reservoir C) sign.

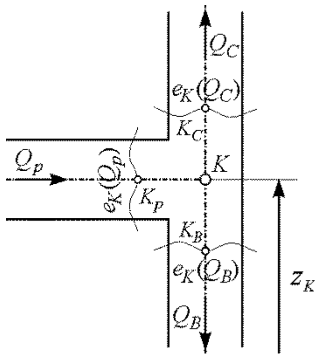
Bernoulli equation from water level in reservoir A to (bifurcation) point K:

$$Y_p = \left(\frac{p_{m.Kp}}{\rho} + \frac{8Q_p^2}{\pi^2 d_1^4} + gz_K \right) + \frac{8Q_p^2}{\pi^2 d_1^4} \left(\lambda_1 \frac{l_1}{d_1} + \Sigma\zeta_1 \right) \qquad (1)$$

Instead of absolute pressures (p_a) on water surface and in bifurcation point K, relative pressures ($p_a + p_{m.Kp}$) are considered.

Also, it is assumed that velocity v_A on water surface of reservoir A is negligibly small, which is usually correct assumption

First term in bracket on the right side of equation (1) represents total specific fluid energy in bifurcation point K, more precisely in point K_p just ahead of bifurcation point, in wich flow is still Q_p (see picture). Overpressure $p_{m.Kp}$ in this point is unknown and height z_K in this point is not even given in description of the problem. On the other hand, knowing this values is not even necessary for solving this problem, so all three terms in bracket can be replaced with only one (extra) unknown – specific energy in intersection just ahead of bifurcation point K:



$$e_K(Q_p) = \frac{p_{m.Kp}}{\rho} + \frac{8Q_p^2}{\pi^2 d_1^4} + gz \quad (2)$$

Putting the equation (2) in equation (1) we get:

$$Y_p = e_K(Q_p) + \frac{8Q_p^2}{\pi^2 d_1^4}\left(\lambda_1 \frac{l_1}{d_1} + \Sigma\zeta_1\right)$$

Equation above should be rearranged so that on the left side only specific energy, in bifucation point, is found $e_K(Q_p)$:

$$e_K(Q_p) = Y_p - \frac{8Q_p^2}{\pi^2 d_1^4}\left(\lambda_1 \frac{l_1}{d_1} + \Sigma\zeta_1\right) = Y_p - \Delta y_{gA-K}(Q_p) = Y_{pr}(Q_p) \quad (3)$$

Function shown on the right side of equation is called “reduced characteristic of the pump” and is labeled as Y_{pr}. That is actually performance characteristic of the pump reduced by hydraulic losses in pump pipeline $\Delta y_{gA\text{-}K}(Q_p)$.

Similarly, Bernoulli equation from bifurcation point K, precisely from point K_B just behind of bifurcation point in which starts flow Q_B (see picture above), to water level in reservoir B, can be written:

$$e_K(Q_B) = gH + \frac{8Q_B^2}{\pi^2 d_2^4}\left(\lambda_2 \frac{l_2}{d_2} + \zeta_B\right) = Y_{cK-B}(Q_B) \quad (4)$$

where $Y_{cK\text{-}B}(Q_B)$, on the right side of equation, has actually the form of system characteristic, i.e. branch of system characteristic K-B:

So, Bernoulli equation from bifurcation point K (K_C on picure) to water level in reservoir C (and vice versa), after rearranging is (characteristic of system branch K-C):

$$e_K(Q_C) = g(H+h) + \frac{8Q_C|Q_C|}{\pi^2 d_2^4}\left(\lambda_2 \frac{l_2}{d_2} + \zeta_C\right) = Y_{cK-C}(Q_C) \quad (5)$$

In equation (5) it is adopted that, $Q_C > 0$ when it’s flowing to reservoir C, $Q_C < 0$ when it’s flowing from reservoir C (see picture). It is assumed that local coefficient ζ_C is same in both flowing directions on pipeline branch K-C.

Finally, continuity equation for junction K is:

$$Q_p = Q_B + Q_C \qquad (6)$$

Pump characteristics $Y_p(Q_p), \eta_p(Q_p)$, reduced pump characteristic $Y_{pr}(Q_p)$ and system characteristics $\Delta y_{gA\text{-}K}(Q_p)$, $Y_{cK\text{-}B}(Q_B)$и $Y_{cK\text{-}C}(Q_C)$ are shown graphically.

In stable working regime of pump system, all left sides of equations (3), (4) и (5) numerically must be the same (specific energy in junction K), so corresponding characteristics in equations on right side must also be the same. Another words, acquired system and reduced characteristics of system can be put in parallel connection.

It is usual to parallel connect system characteristics $K\text{-}B$ и $K\text{-}C$, following equation of continuity (6), at same head / specific energy (e_K) flows are being summed Q_B and Q_C. Result, system parallel characteristic $K\text{-}B$ and $K\text{-}C$ shown in picture → $Y_{cB+C}(Q_B + Q_C) = Y_{cB+C}(Q_p)$.

In inter-section of system parallel characteristic $Y_{cB+C}(Q_p)$ and reduced pump characteristic $Y_{pr}(Q_p)$ operational system point is acquired RT_1.

It can be assumed that operational point RT_1 is found at inter-section of some imaginary pump characteristic (which consists of real pump characteristic and system characteristic $A\text{-}K$) and imaginary system characteristic (which consists of system characteristics $K\text{-}B$ и $K\text{-}C$). In this way, problem is reduced to system of one pump which is working in simple system (1st exercise).

From operational point RT_1, inflows to reservoirs B and C can be found graphically. (system characteristics $Y_{cK\text{-}B}$ и $Y_{cK\text{-}C}$ horizontally $e_K = e_{RT1} = const$).

$$RT_1 \rightarrow \qquad Q_{B1} = 16{,}9\,\frac{\text{L}}{\text{s}} \qquad Q_{C1} = 9{,}2\,\frac{\text{L}}{\text{s}}$$

Operational point of the pump RT_{p1} can be found on performance characteristic of the pump $Y_p(Q_p)$, above operational point RT_1 (on the vertical $Q_{p1} = Q_{RT1} = const$). Vertical distance of operational point of pump RT_{p1} and operational system point RT_1 equals to hydraulic losses in pump system at flow– $\Delta y_{gA\text{-}K}(Q_{p1})$.

From operational pump point RT_{p1} head, flow and efficiency coefficient can be acquired, so finally required shaft power can be calculated:

$$RT_1 \rightarrow RT_{p1} \rightarrow \qquad Y_{p1} = 402{,}9\,\frac{\text{J}}{\text{kg}} \qquad Q_{p1} = 26{,}1\,\frac{\text{L}}{\text{s}} \qquad \eta_{p1} = 73{,}2\ \%$$

$$P_{p1} = \frac{\rho Q_{p1} Y_{p1}}{\eta_{p1}} = 14{,}4\ \text{kW}$$

Solution (2) – balancing the flow:

Operational system point RT_2 is obtained from next conditions:

$$Q_{B2} = Q_{C2} = Q_{p2}/2 \quad \text{при} \quad e_K(Q_{p2}) = e_K(Q_{B2}) = e_K(Q_{C2}) \qquad (7)$$

Graphical solution of the problem comes down to, finding the point M on system characteristic $Y_{cK\text{-}C}(Q_C)$, which is at same distance from Y ordinate as from reduced pump characteristic $Y_{pr}(Q_p)$, i.e. from point in which condition (7) is satisfied. New system characteristic *K-B* (with regulating valve) should go through that point *K-B.* New parallel system characteristic *K-B* и *K-C* intersects reduced pump characteristic in operational point RT_2 (both new system characteristics are marked with interrupted green line on graph).

From point *M* it is acquired:

$$Q_{B2} = Q_{C2} = 12{,}2\,\frac{\text{L}}{\text{s}}$$

$$e_K(Q_{B2}) = e_K(Q_{C2}) = e_K(Q_{p2}) = 367{,}3\,\frac{\text{J}}{\text{kg}}$$

By putting this solution in equation (4) and solving it by unknown local resistance coefficient, finally, local resistance coefficient of regulating valve is calculated:

$$\zeta_{B2} = \frac{\pi^2 d_2^4 \left[e_{Km}(Q_{B2}) - gH\right]}{8Q_{B2}^2} - \lambda_2 \frac{l_2}{d_2} = 14{,}5$$

Solution (3):

From condition that Q_{C3} = 0, ie Q_{B3} = Q_{p3} system operational point is obtained RT_3, shown on graph. Using equation (3) operational pump point RT_{p3}, at unknown rotational speed n_3, has to be above operational system point for losses in pump system $\Delta y_{gA\text{-}K}$ at flow Q_{B3} = Q_{p3}. From that operational point RT_{p3} required pump parameters can be read:

$$RT_{p3} \rightarrow \qquad Y_{p3} = 314\,\frac{\text{J}}{\text{kg}} \qquad Q_{p3} = 14{,}1\,\frac{\text{L}}{\text{s}}$$

To find unknown rotational speed, “similar parable” should go through operational pump point RT_{p3}, with next form (see picture):

$$Y_s(Q_S) = \left(\frac{Y_{p3}}{Q_{p3}^2}\right) Q_s^2$$

In intersection of similar parable $Y_s(Q_s)$ and performance characteristic of the pump $Y_p(Q_p)$ at n = 1450 rpm, similar point ST_3 is obtained, from whom head, flow and efficiency coefficient can be read:

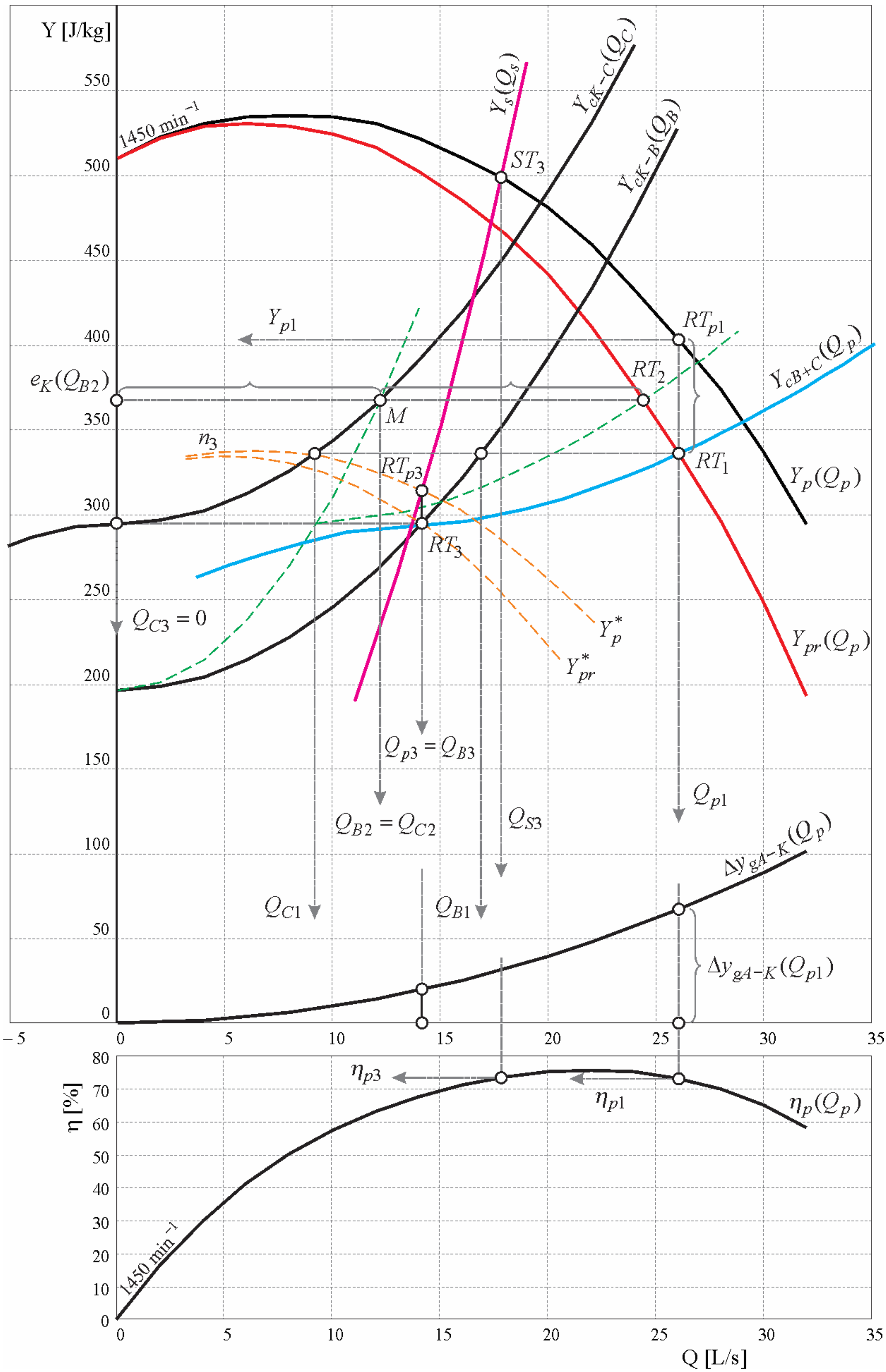
$$ST_3 \rightarrow \qquad Q_{s3} = 17{,}8\,\frac{\text{L}}{\text{s}} \qquad \eta_{s3} = \eta_{p3} = 73{,}3\ \%$$

Required rotational speed is acquired from affinity laws:

$$n_3 = \frac{Q_{p3}}{Q_{s3}}\, n = 1151\ \text{min}^{-1}$$

Finally required shaft power is:

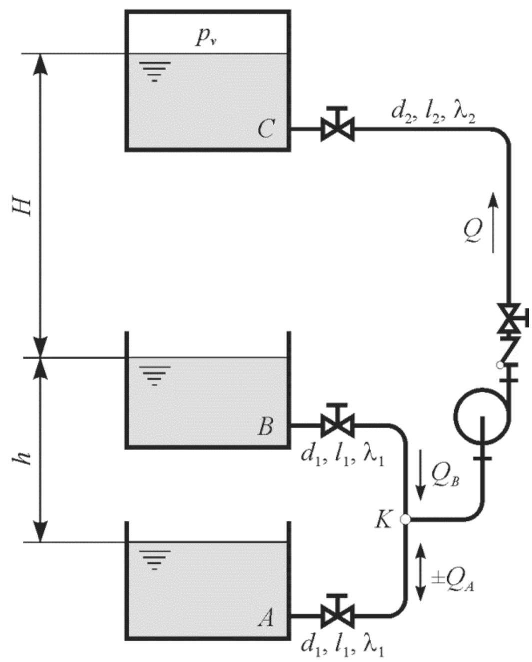
$$P_{p3} = \frac{\rho Q_{p3} Y_{p3}}{\eta_{p3}} = 6{,}1\ \text{kW}$$


Y [J/kg]
550
500
450
400
350
300
250
200
150
100
50
0
1450 min⁻¹
$Y_s(Q_s)$
$Y_{cK-C}(Q_C)$
$Y_{cK-B}(Q_B)$
ST_3
Y_{p1}
RT_{p1}
RT_2
$e_K(Q_{B2})$
M
$Y_{cB+C}(Q_p)$
n_3
RT_{p3}
RT_1
$Y_p(Q_p)$
RT_3
$Q_{C3} = 0$
Y_p^*
Y_{pr}^*
$Y_{pr}(Q_p)$
$Q_{p3} = Q_{B3}$
Q_{p1}
$Q_{B2} = Q_{C2}$
Q_{S3}
$\Delta y_{gA-K}(Q_p)$
Q_{C1}
Q_{B1}
$\Delta y_{gA-K}(Q_{p1})$
– 5
0
5
10
15
20
25
30
35
80
70
60
50
40
30
20
10
0
η [%]
η_{p3}
η_{p1}
$\eta_p(Q_p)$
1450 min⁻¹
Q [L/s]

Homework:

1. Find local loss coefficients of regulating valves on branches *KB* and *KC* so that inflows to reservoirs *B* and *C* are the same and that pump is working with max efficiency coefficient.
2. Find rotational speed of pump (regulating valves on branches *KB* are *KC* fully open) at which reservoir *C* is emptying with flow 2 L/s (negative flow)? What is the shaft power required then?

Exercise 3: Centrifugal pump with known performance characteristics at n = 1450 rpm, transfers water from reservoirs A and B to reservoir C through pipeline shown on figure. Water level differences are h = 5 m and H = 14 m. Reservoir C is closed and at under-pressure p_v = 0,5 bar. Pipeline consists of two equal branches AK and BK whose characteristics are d_1 = 80 mm, l_1 = 18 m, λ_1 = 0,025, $\Sigma\zeta_1$ = 4 and main pipeline KC with characteristics d_2 = 125 mm, l_2 = 50 m, λ_2 = 0,022 и $\Sigma\zeta_2$ = 5.



Find:

1. Outflows from reservoirs A and B, flow, head and required shaft power of the pump.
2. Find rotational speed of the pump if inflow to reservoir C is raised for 20 %? What is the shaft power required then ?
3. At which rotational speed will outflow form reservoir A stop (no flow)? What shaft power is required then?

Performance characteristics of the pump at n = 1450 rpm:

Q_p [L/s]	0	5	10	15	20	25	30	35	40
Y_p [J/kg]	284,5	304	319	324	314	284,5	235,5	167	78,5
η_p [%]	0	30	50	63	71	75	75	70	58

Solution (1):

Problem is solved using Bernoulli equations for all parts of pipeline in which only one unknown flow figures and using continuity equation for bifurcation point K.

If it is adopted that ground level is on water surface in reservoir A, all height differences are measured from water level in reservoir A, and that velocities of water on surfaces in reservoirs are negligible, appropriate Bernoulli equations can be written:

B.E. A-K ($Q_A > 0$) and K-A ($Q_A < 0$)

$$0 = e_K(Q_A) + \frac{8Q_A|Q_A|}{\pi^2 d_1^4}\left(\lambda_1 \frac{l_1}{d_1} + \sum\zeta_1\right)$$

rearranging by $e_K(Q_A)$

$$e_K(Q_A) = -\frac{8Q_A|Q_A|}{\pi^2 d_1^4}\left(\lambda_1 \frac{l_1}{d_1} + \sum\zeta_1\right) = Y_{cA-K}(Q_A) \qquad (1)$$

which has a form of system characteristic, branch system characteristic A-K.

B.E. *B-K*

$$gh = e_K(Q_B) + \frac{8Q_B^2}{\pi^2 d_1^4}\left(\lambda_1 \frac{l_1}{d_1} + \sum \zeta_1\right)$$

rearranging by $e_K(Q_B)$

$$e_K(Q_B) = gh - \frac{8Q_B^2}{\pi^2 d_1^4}\left(\lambda_1 \frac{l_1}{d_1} + \sum \zeta_1\right) = Y_{cB-K}(Q_B) \qquad (2)$$

which represents system characteristic of branch *B-K*.

B.E. *K-C*

$$e_K(Q) + Y_p = g(h+H) - \frac{p_v}{\rho} + \frac{8Q^2}{\pi^2 d_2^4}\left(\lambda_2 \frac{l_2}{d_2} + \Sigma\zeta_2\right) = Y_c(Q) \qquad (3)$$

Continuity equation for bifurcation point K:

$$Q = Q_A + Q_B \qquad (4)$$

Term on right side of the equation (3) represents system characteristic of the main pipeline to reservoir *C* ie. $Y_c(Q)$. Term on the right side represents serial connection of pump and pump (suction) pipeline which in this case consists of pipeline parts *A-K* и *B-K*. This connection can be formally called a reduced pump characteristic:

$$Y_{pr}(Q) = e_K(Q) + Y_p \qquad (5)$$

Equation (3) can now be written as:

$$Y_{pr}(Q) = Y_c(Q) \qquad (6)$$

So this relatively complex hydraulic system can be reduced to simple system which consists of main pipeline and one imaginary pump, which characteristic consists of real pump characteristic and suction pipeline characteristic.

Pump characteristics $Y_p(Q_p), \eta_p(Q_p)$, reduced pump characteristic $Y_{pr}(Q)$, suction pipelines characteristics $Y_{cA-K}(Q_A)$, $Y_{cB-K}(Q_B)$ and main pipeline characteristic $Y_c(Q)$ are shown graphically. Notice that $Q_p = Q$.

Now only function $e_K(Q)$ is set to be defined, in equation (3) or (5) she represents specific energy in bifurcation point *K*, ie in section just behind bifurcaion point, in which flow *Q* is governing. Same specific energy is shown in equation (1) in function of flow Q_A and in equation (2) in function of flow Q_B. In stable regime left sides of equations (1) and (2) numerically must be the same, so appropriate characteristics on the left sides must be the same. Another words, characteristics of pipe branches $Y_{cA-K}(Q_A)$ и $Y_{cB-K}(Q_B)$ should be put in parallel connection following continuity equation (4), i.e. at same head/specific energy (e_K) flows are counted Q_A and Q_B.

Result, parallel characteristic of suction branches *A-K* и *B-K* is shown on graph→ $Y_{cA+B}(Q_A + Q_B) = Y_{cA+B}(Q) = e_K(Q)$.

Following equation (5), by summing acquired parallel characteristic with pump characteristic, reduced pump characteristic is acquired and shown on graph → $Y_{pr}(Q)$.

Finally, using equation (6), in intersection of main pipeline characteristic $Y_c(Q)$ and reduced pump characteristic $Y_{pr}(Q)$ operational system point is acquired RT_1, from whom flow through main pipeline can be read i.e. inflow to reservoir C:

$$RT_1 \rightarrow \qquad Q_1 = 32\frac{\text{L}}{\text{s}}$$

Flows from reservoirs A и B are acquired by going backwards through procedure which led to reduced pump characteristic $Y_{pr}(Q)$ and operational system point RT_1: from operational point RT_1 vertical line is drawn $Q_1 = const$ to parallel characteristic $Y_{cA+B}(Q) = e_K(Q)$, and then from acquired intersection point, horizontal line is drawn $e_K(Q_1) = const$ to suction branches characteristics $Y_{cA\text{-}K}(Q_A)$ and $Y_{cB\text{-}K}(Q_B)$ from which flows can be read:

$$Q_{A1} = 12{,}0\frac{\text{L}}{\text{s}} \qquad Q_{B1} = 20{,}0\frac{\text{L}}{\text{s}}$$

Operational pump point RT_{p1} is acquired on pump characteristic $Y_p(Q_p)$, above operational system point RT_1 (on vertical line $Q_{p1} = Q_1 = const$). Vertical distance from operational pump point RT_{p1} to operational system point RT_1 equals to specific energy in bifurcation point K at acquired flows → $e_K(Q_1) = e_K(Q_{A1}) = e_K(Q_{B1})$.

From operational pump point RT_{p1} flow, head and efficiency coefficient of the pump can be read from whom required shaft power can be counted:

$$RT_1 \rightarrow RT_{p1} \rightarrow \qquad Y_{p1} = 210{,}6\frac{\text{J}}{\text{kg}} \qquad Q_{p1} = Q_1 = 32{,}0\frac{\text{L}}{\text{s}} \qquad \eta_{p1} = 73{,}7\ \%$$

$$P_{p1} = \frac{\rho Q_{p1} Y_{p1}}{\eta_{p1}} = 9{,}15\ \text{kW}$$

Solution (2) –regulating the pump at desired flow by changing rotational speed of the pump

Operational system point RT_2 is acquired on system characteristic $Y_c(Q)$ from requirement that flow Q_2 is for 20% greater than Q_1:

$$Q_2 = 1{,}2 Q_1 = 1{,}2 \cdot 32\ \text{L/s} = 38{,}4\ \text{L/s} \quad \rightarrow \quad RT_2$$

Similarly as in solution (1), reduced pump characteristic Y_{pr}^* must go through operational system point RT_2, that characteristic would be equal as pump chacteristic Y_p^* at unknown rotational speed of the pump n_2. Pump characteristic Y_p^* itself should go through operational pump point RT_{p2} which, following equation (5), must be above operational system point RT_2 for value of specific energy in bifurcation point K at demanded flow of the system → $e_K(Q_2)$. From operational pump point RT_{p2} working parameters of the pump can be read at still unknown rotational speed n_2:

$$RT_{p2} \rightarrow \qquad Y_{p2} = 251{,}8\frac{\text{J}}{\text{kg}} \qquad Q_{p2} = Q_2 = 38{,}4\ \text{L/s}$$

To find unknown rotational speed n_2, similarity parable must pass through operational pump point RT_{p2} (see graph):

$$Y_s^*(Q_s) = \left(\frac{Y_{p2}}{Q_{p2}^2}\right)Q_s^2 = 0{,}1707Q_s^2$$

In intersection of similarity parable $Y_s^*(Q_s)$ and working pump performance characteristic $Y_p(Q_p)$ at n = 1450 min^{-1}, similar point ST_2 is acquired, from whom flow (or head) and efficiency coefficient can be read:

$$ST_2 \rightarrow \qquad Q_{s2} = 33{,}4\frac{\mathrm{L}}{\mathrm{s}} \qquad \eta_{s2} = \eta_{p2} = 72{,}2\ \%$$

Required rotational speed of the pump can be obtained from affinity laws:

$$n_2 = \frac{Q_{p2}}{Q_{s2}}n = 1666\ \mathrm{min}^{-1}$$

Finally required shaft power of the pump is:

$$P_{p2} = \frac{\rho Q_{p2} Y_{p2}}{\eta_{p2}} = 13{,}4\ \mathrm{kW}$$

Solution (3):

From requirement that Q_{A3} = 0, i.e. $Q_{B3} = Q_3$, operational system point RT_3 is obtained on system characteristic $Y_c(Q)$. Reduced pump characteristic Y_{pr}^{**} that suits pump characteristic Y_p^{**} must go through operational system point RT_3 at unknown rotational n_3, and pump characteristic iself Y_p^{**} , regarding that now the value of specific energy in bifurcation point K equals zero ie $e_K(Q_3) = 0$.

Another words, operational pump point RT_{p3} is in this case at same spot as operational system point RT_3.

From that operational pump point RT_{p3} required working parameters of the pump can be read:

$$RT_{p3} \rightarrow \qquad Y_{p3} = 148{,}0\frac{\mathrm{J}}{\mathrm{kg}} \qquad Q_{p3} = Q_3 = 16{,}0\frac{\mathrm{L}}{\mathrm{s}}$$

Unknown rotational speed of the pump is acquired using affinity laws, i.e. similarity parable must go through operational pump point RT_{p3} (see graph):

$$Y_s^{**}(Q_s) = \left(\frac{Y_{p3}}{Q_{p3}^2}\right)Q_s^2$$

In intersection of so acquired similarity parable $Y_s^{**}(Q_s)$ and pump performance characteristic $Y_p(Q_p)$ at n = 1450 min^{-1}, similar point ST_3 is acquired, from whom flow (or head) and efficiency coefficient can be read:

$$ST_3 \rightarrow \qquad Q_{s3} = 22{,}8\frac{\mathrm{L}}{\mathrm{s}} \qquad \eta_{s3} = \eta_{p3} = 73{,}7\ \%$$

Demanded rotational speed of the pump and acquired from affinity laws:

$$n_3 = \frac{Q_{p3}}{Q_{s3}} n = 1118 \text{ min}^{-1}$$

Finally required shaft power of the pump is:

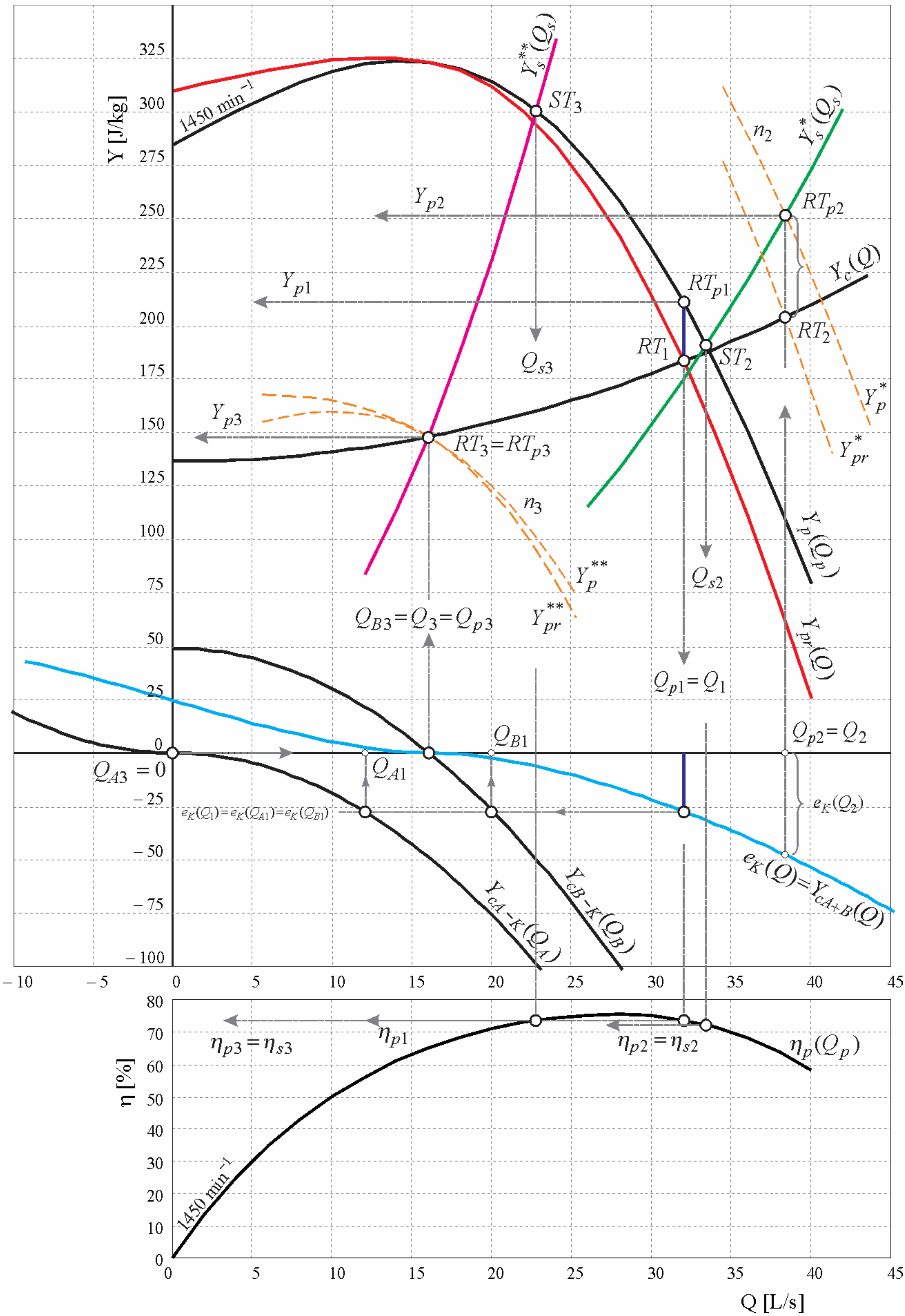
$$P_{p3} = \frac{\rho Q_{p3} Y_{p3}}{\eta_{p3}} = 3{,}2 \text{ kW}$$

Homework:

1. What are the outflows from reservoirs *A* и *B* in second problem (2)?
2. Solve same exercise by writing Bernoulli equation (3) in this form:

$$Y_p = g(h+H) - \frac{p_v}{\rho} + \frac{8Q^2}{\pi^2 d_2^4}\left(\lambda_2 \frac{l_2}{d_2} + \Sigma\zeta_2\right) - e_K(Q) = Y_c(Q) - e_K(Q)$$

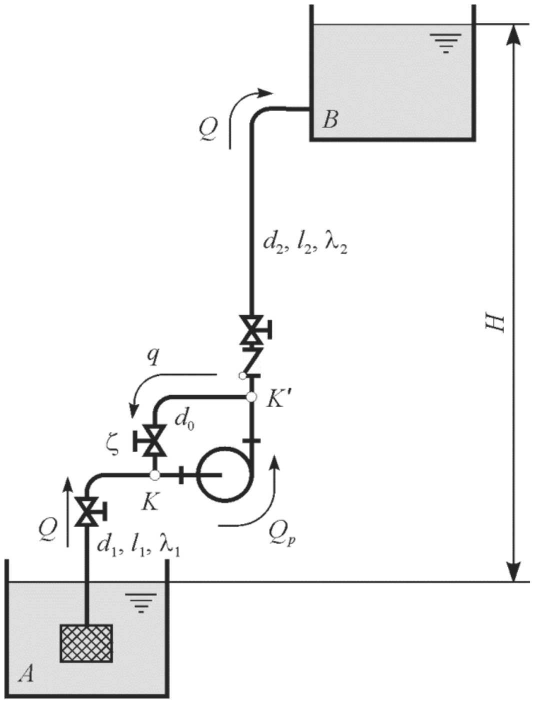
   i.e., main pipeline characteristic (system characteristic) $Y_c(Q)$ and parallel characteristic of suction pipeline $e_K(Q) = Y_{cA+B}(Q)$ must be put in serial connection.



Y [J/kg]
325
300
275
250
225
200
175
150
125
100
75
50
25
0
– 25
– 50
– 75
– 100
1450 min⁻¹
$Y_s^{**}(Q_s)$
ST_3
$Y_s^{*}(Q_s)$
n_2
Y_{p2}
RT_{p2}
$Y_c(Q)$
Y_{p1}
RT_{p1}
RT_2
RT_1
ST_2
Q_{s3}
Y_{p3}
$RT_3 = RT_{p3}$
Y_p^{*}
Y_{pr}^{*}
n_3
$Y_p(Q_p)$
Y_p^{**}
Q_{s2}
$Q_{B3} = Q_3 = Q_{p3}$
Y_{pr}^{**}
$Y_{pr}(Q)$
$Q_{p1} = Q_1$
Q_{B1}
$Q_{p2} = Q_2$
$Q_{A3} = 0$
Q_{A1}
$e_K(Q_1) = e_K(Q_{A1}) = e_K(Q_{B1})$
$e_K(Q_2)$
$e_K(Q) = Y_{cA+B}(Q)$
$Y_{cA-K}(Q_A)$
$Y_{cB-K}(Q_B)$
– 10
– 5
0
5
10
15
20
25
30
35
40
45
η [%]
80
70
60
50
40
30
20
10
0
$\eta_{p3} = \eta_{s3}$
η_{p1}
$\eta_{p2} = \eta_{s2}$
$\eta_p(Q_p)$
1450 min⁻¹
Q [L/s]

Exercise 4: Centrifugal pump with known characteristics at n = 2900 rpm works in a system shown on figure and pumps water from reservoir A to reservoir B. Just at the discharge point on the pump bypass is set, through whom part of the water is being returned to the suction part, when valve at the bypass section is open.

Diameters of suction and discharge pipelines are the same $d_1 = d_2 = d$ = 125 mm. Length of the pipeline is $l_1 + l_2 = L$ = 650 m and friction coefficient of all sections of pipeline is $\lambda_1 = \lambda_2 = \lambda$ = 0,023, total local resistance is $\Sigma\zeta_1 + \Sigma\zeta_2 = \Sigma\zeta$ =15. Diameter of bypass is d_0 = 50 mm. Difference in water level height in reservoirs is H = 28 m



Find:

1. Local coefficient of the valve ζ on bypass, if the inflow to reservoir B is same as flow through bypass?

   What shaft power is required then?

2. For local coefficient of the valve at (1) ζ find flow, head and required shaft power of the pump if the pump rotates at n_2 = 2700 min^{-1}.

   What is the inflow to reservoir B then?

3. Find local coefficient of the valve ζ on the bypass, if the pump is working with max efficiency (at n_2 = 2700 rpm)?

Performance characteristics of the pump at n = 2900 rpm:

Q_p [L/s]	0	4	8	12	16	20	24	28	32	36
Y_p [J/kg]	515	530	535	530	512	480	432	373	295	187
η_p [%]	0	30	50	63	71	75	75	70	58	36

Solution (1): Regulating the flown using bypass valve

Problem is solved using Bernoulli equation for all parts of the pipeline where only one unknown flow figures (Q, Q_p or q) and continuity equation for bifurcation point K or K'.

B.E. *A*-*K*

$$0 = e_K(Q) + \frac{8Q^2}{\pi^2 d_1^4}\left(\lambda_1 \frac{l_1}{d_1} + \sum \zeta_1\right) \tag{1}$$

B.E. *K'-B*

$$e_{K'}(Q) = gH + \frac{8Q^2}{\pi^2 d_2^4}\left(\lambda_2 \frac{l_2}{d_2} + \sum \zeta_2\right) \tag{2}$$

In equations (1) and (2) figures only one flow so they can be added up, so after rearranging:

$$e_{K'}(Q) - e_K(Q) = \Delta e(Q) = gH + \frac{8Q^2}{\pi^2 d_1^4}\left(\lambda_1 \frac{l_1}{d_1} + \sum \zeta_1\right) + \frac{8Q^2}{\pi^2 d_2^4}\left(\lambda_2 \frac{l_2}{d_2} + \sum \zeta_2\right)$$

Taking into account that $d_1 = d_2 = d$, $l_1 + l_2 = L$, $\lambda_1 = \lambda_2 = \lambda$ and $\Sigma\zeta_1 + \Sigma\zeta_2 = \Sigma\zeta$ equation above can be written in more simple way:

$$\Delta e(Q) = gH + \frac{8Q^2}{\pi^2 d^4}\left(\lambda \frac{L}{d} + \sum \zeta\right) = Y_c(Q) \tag{3}$$

where term on right side represents system characteristic from reservoir *A* to reservoir *B*.

B.E. *K-K'*

$$e_K(Q_p) + Y_p = e_{K'}(Q_p)$$

Hydraulic losses in short pipeline parts are negligible small. By rearranging this equation so that on left side stays only difference of specific energies in bifurcation points *K'* and *K* we get:

$$\Delta e(Q_p) = e_{K'}(Q_p) - e_K(Q_p) = Y_p \tag{4}$$

what is actually definition of the head of the pump.

B.E. *K'-K*

$$e_{K'}(q) = e_K(q) + \frac{8q^2}{\pi^2 d_0^4}\zeta$$

where friction losses at the bypass are negligible small compared to local losses on valve at the bypass. By rearranging this equation at the same way as in equation (3) and (4) it is obtained:

$$\Delta e(q) = e_{K'}(q) - e_K(q) = \frac{8q^2}{\pi^2 d_0^4}\zeta = Y_{bps}(q) \tag{5}$$

what represents bypass characteristic.

Continuity equation for bifurcation point *K* is:

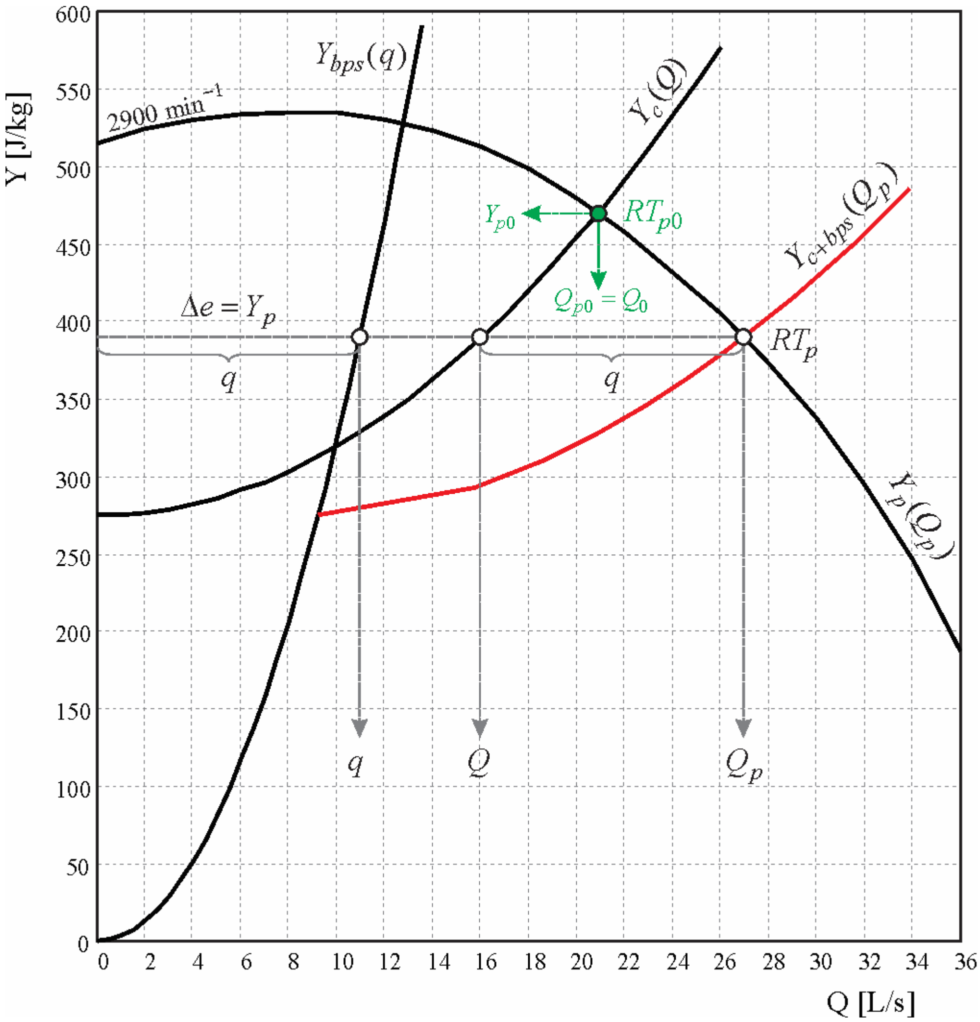
$$Q_p = Q + q \tag{6}$$

and for bifurcation point *K'*:

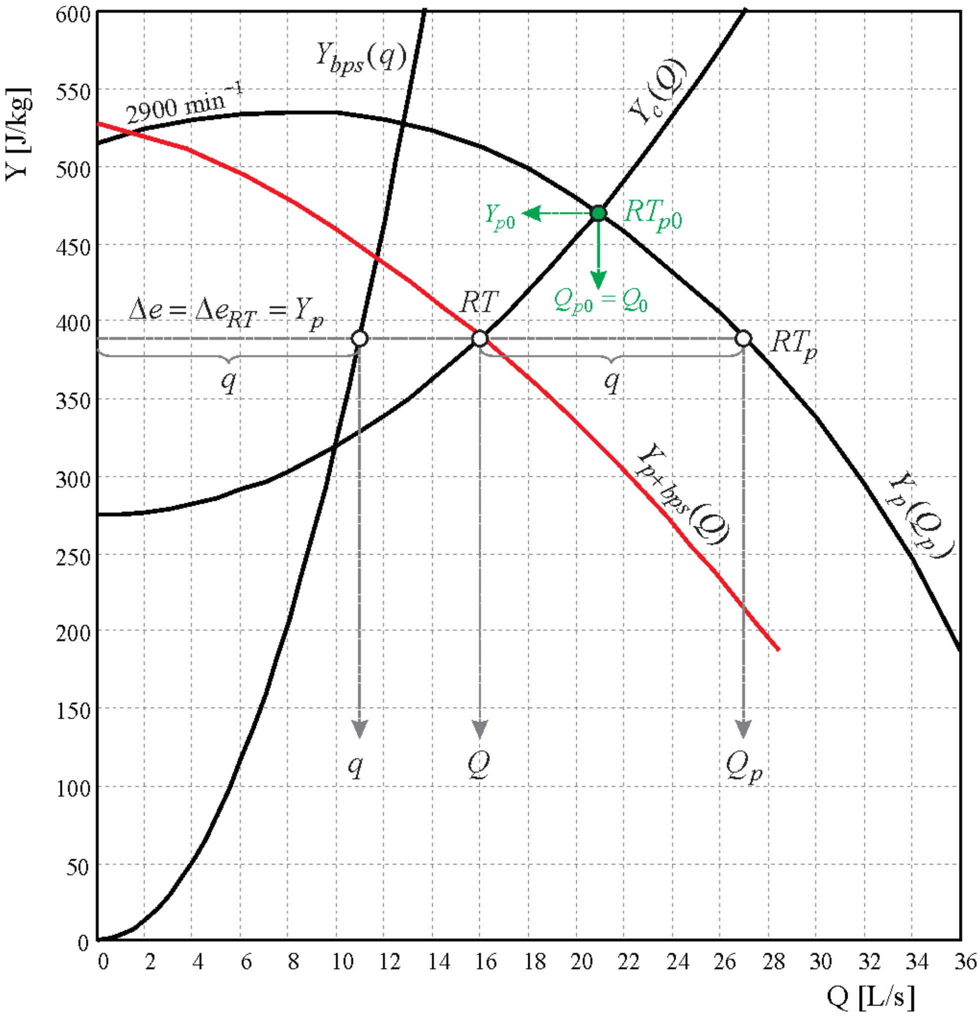
$$Q = Q_p - q \tag{6*}$$

Equations (6) and (6*) are mathematically identical, i.e. they represent one equation written in two formally different forms. Depending on which form is used (6) or (6*) graphical solution can be found in two ways. In stable regime, left sides of equations (3), (4) and (5) numerically must be the same i.e. $\Delta e(Q) = \Delta e(Q_p) = \Delta e(q)$ which points to parallel connection of corresponding parts.

If form (6) of continuity equation is used, parallel connection should be made from system characteristic and bypass characteristic → $Y_{c+bps}(Q + q) = Y_{c+bps}(Q_p)$. In intersection of obtained parallel characteristic $Y_{c+bps}(Q_p)$ with pump characteristic $Y_p(Q_p)$ operational pump point RT_p is obtained, from whom flow and head of the pump can be read (Q_p, Y_p). Flows through pipeline Q and bypass q are obtained from intersection points on horizontal line $\Delta e = Y_p = const$ with system characteristic $Y_c(Q)$, ie bypass characteristic $Y_{bps}(q)$. This solution is shown on graph below, it is assumed that local coefficient of the valve at bypass is known and is $\zeta = 25$.



If form (6*) of continuity equation is used, parallel connection should be made from pump characteristic and bypass characteristic, at same head (Δe) flow of the pump is reduced for the flow in the bypass од → $Y_{p+bps}(Q_p - q) = Y_{p+bps}(Q)$. In intersection of acquired parallel characteristic $Y_{p+bps}(Q)$ with system characteristic $Y_c(Q)$ operatioanl system point is acquired RT from whom flow Q through pipeline can be read. Flow through bypass q and operational pump point RT_p are obtained from intersection points on horizontal line $\Delta e = \Delta e_{RT} = const$ with bypass characteristic $Y_{bps}(q)$ and pump characteristic $Y_p(Q_p)$. From operational pump point flow and head of the pump can be read (Q_p, Y_p), at $Y_p = \Delta e$. This solution is shown on graph below, it is assumed that local coefficient of the valve at bypass is known and is $\zeta = 25$.



In first case, in parallel connection between system characteristic and bypass characteristic, imaginary system characteristic is acquired, in whose intersection with pump characteristic operational pump point is acquired. In second case, in parallel connection between pump characteristic and bypass characteristic imaginary pump characteristic is obtained, in whose intersection with system characteristic operational point of system is found. In both cases, it can be said that this problem is simplified to simple pipeline (exercise 1).

It should be noticed that in case that valve on bypass is closed ($q = 0$) operational point is found in intersection of pump characteristic and system characteristic (RT_{p0} on both graphs). Now it is clear that by opening bypass (valve) flow through pipeline is in decline ($Q_0 \rightarrow Q$) and through pump in incline ($Q_{p0} \rightarrow Q_p$). On the other hand, for smaller flow to reservoir B ($Q < Q_0$) pump is working with smaller head ($Y_p < Y_{p0}$).

Regulating flow using bypass is found in practice (not so often), in case of big pumps in thermo energetic plants.

In this particular problem, local valve coefficient on bypass is unknown and can be found from next term:

$$Q_1 = q_1 = Q_{p1}/2 \quad \text{при} \quad \Delta e(Q_{p1}) = \Delta e(Q_1) = \Delta e(q_1) \qquad (7)$$

Graphical solution of the problem comes down to finding operational system point RT_1 on system characteristic $Y_c(Q)$ that is at same distance from Y axis and reduced pump characteristic $Y_{pr}(Q_p)$, i.e. point in which term (7) is satisfied (see graph). Bypass characteristic $Y_{bps1}(q)$ should go through operational system point RT_1. If for solving problem, continuity equation (6*) is used, parallel characteristic of the pump and bypass $Y_{p+bps1}(Q)$ will also go through that point, which is also shown on graph. Operational pump point RT_{p1} is found in intersection of horizontal line $\Delta e_{RT1} = const$ and pump characteristic $Y_p(Q_p)$.

From operational point RT_1 it can be read:

$$RT_1 \rightarrow \qquad q_1 = Q_1 = 14{,}2 \ \frac{\text{L}}{\text{s}}$$

$$\Delta e(q_1) = \Delta e(Q_1) = \Delta e(Q_{p1}) = e_{RT1} = 365{,}1 \ \frac{\text{J}}{\text{kg}}$$

By changing values in equation (5) and solving it by unknown local coefficient, solution is acquiered:

$$\zeta_1 = \frac{\pi^2 d_0^4 \Delta e(q_1)}{8 q_1^2} = 13{,}9$$

Bypass characteristic $Y_{bps}(q)$ for this value of local coefficient is shown on graph.

From operational pump point RT_{p1} flow, head and efficiency coefficient can be read, using them required shaft power can be found:

$$RT_{p1} \rightarrow \qquad Y_{p1} = 365{,}1 \ \frac{\text{J}}{\text{kg}} \qquad Q_{p1} = 28{,}4 \ \frac{\text{L}}{\text{s}} \qquad \eta_{p1} = 69{,}0 \ \%$$

$$P_{p1} = \frac{\rho Q_{p1} Y_{p1}}{\eta_{p1}} = 15{,}1 \ \text{kW}$$

Solution (2):

Pump performance characteristics $Y_p(Q_p)$ at n = 2900 rpm should be recalculated for new rotational speed n_2 = 2700 rpm using affinity laws:

$$Q_p^* = \frac{n_2}{n} Q_p$$

$$Y_p^* = \left(\frac{n_2}{n}\right)^2 Y_p$$

$$\eta_p^* = \eta_p$$

Results are shown in table.

Pump performance characteristics at n = 2900 rpm:

Q_p [L/s]	0	4	8	12	16	20	24	28	32	36
Y_p [J/kg]	515	530	535	530	512	480	432	373	295	187
η_p [%]	0	30	50	63	71	75	75	70	58	36

Q_p: $\times \frac{2700}{2900}$; Y_p: $\times \left(\frac{2700}{2900}\right)^2$; η_p: $=$

Pump performance characteristics at n_2 = 2700 rpm:

Q_p^* [L/s]	0	3,72	7,45	11,17	14,9	18,6	22,35	26,07	29,8	33,52
Y_p^* [J/kg]	446,4	459,4	463,8	459,4	443,8	416,1	374,5	323,3	255,7	162,1
η_p^* [%]	0	30	50	63	71	75	75	70	58	36

Pump performance characteristics $Y_p^*(Q_p^*)$ и $\eta_p^*(Q_p^*)$ at new rotational speed n_2 = 2700 rpm are shown on graph (blue line). Parallel characteristic from new pump characteristic and bypass characteristic is acquired using continuity equation (6*), is also shown on graph (red line) $\rightarrow Y_{p^*+bps1}(Q_p - q) = Y_{p^*+bps1}(Q)$.

In intersection of parallel characteristic $Y_{p^*+bps1}(Q)$ with system characteristic $Y_c(Q)$ operational system point is acquired RT_2 from whom flow can be read. Q_2. Flow through bypass q_2 and operational pump point RT_{p2} are acquired in intersection of horizontal line $\Delta e = \Delta e_{RT2} = const$ with bypass characteristic $Y_{bps1}(q)$ and pump characteristic $Y_p^*(Q_p^*)$.

$$RT_2 \rightarrow \qquad Q_2 = 11{,}7\ \frac{\text{L}}{\text{s}} \qquad q = 13{,}6\ \frac{\text{L}}{\text{s}}$$

$$RT_2 \rightarrow RT_{p2} \rightarrow \qquad Y_{p2} = 333{,}2\ \frac{\text{J}}{\text{kg}} \qquad Q_{p2} = 25{,}3\ \frac{\text{L}}{\text{s}} \qquad \eta_{p2} = 71{,}6\ \%$$

$$P_{p2} = \frac{\rho Q_{p2} Y_{p2}}{\eta_{p2}} = 11{,}85\ \text{kW}$$

Solution (3): Optimization of the pump

Under the terms of the problem pump should work in the optimal point with the max efficiency at n_2 = 2700 rpm. On efficiency characteristic of the pump $\eta_p^*(Q_p^*)$ at max efficiency, coefficient and flow are:

$$\eta_{p3} = \eta_{\max} = 75{,}5\ \% \ \rightarrow \qquad Q_{p3} = 20{,}5\ \frac{\text{L}}{\text{s}}$$

i.e. pump must work in operational system point RT_3 on pump characteristic $Y_p^*(Q_p^*)$:

$$RT_{p3} \rightarrow \qquad Y_{p3} = 396{,}3\ \frac{\text{J}}{\text{kg}} \qquad Q_{p3} = 20{,}5\ \frac{\text{L}}{\text{s}}$$

On the other hand, system characteristic $Y_c(Q)$ doesn't intersect pump characteristic $Y_p^*(Q_p^*)$ in point RT_{p3} but in point RT_0 , system would work in this point if valve at bypass is closed. So it is clear that this valve must be partly open, to fulfill demand that this pump works in RT_{p3}, i.e. with max efficiency.

When valve is partly open i.e. in terms when pump and bypass are in parallel (continuity equation 6*), operational system point RT_3 is found in intersection with horizontal line:

$$\Delta e(Q_{p3}) = \Delta e(Q_3) = \Delta e(q_3) = Y_{p3} = 396{,}3 \text{ J/kg}$$

and system characteristic $Y_c(Q)$. From so acquired system point, flow through pipeline can be read:

$$RT_3 \rightarrow \qquad Q_3 = 16{,}5\ \frac{\text{L}}{\text{s}}$$

so required flow through bypass is:

$$q_3 = Q_{p3} - Q_3 = 4\ \frac{\text{L}}{\text{s}}$$

By exchanging values q_3 and $\Delta e(q_3)$ in equation (5) and solving by unknown coefficient, result is:

$$\zeta_3 = \frac{\pi^2 d_0^4 \Delta e(q_3)}{8 q_3^2} = 189$$

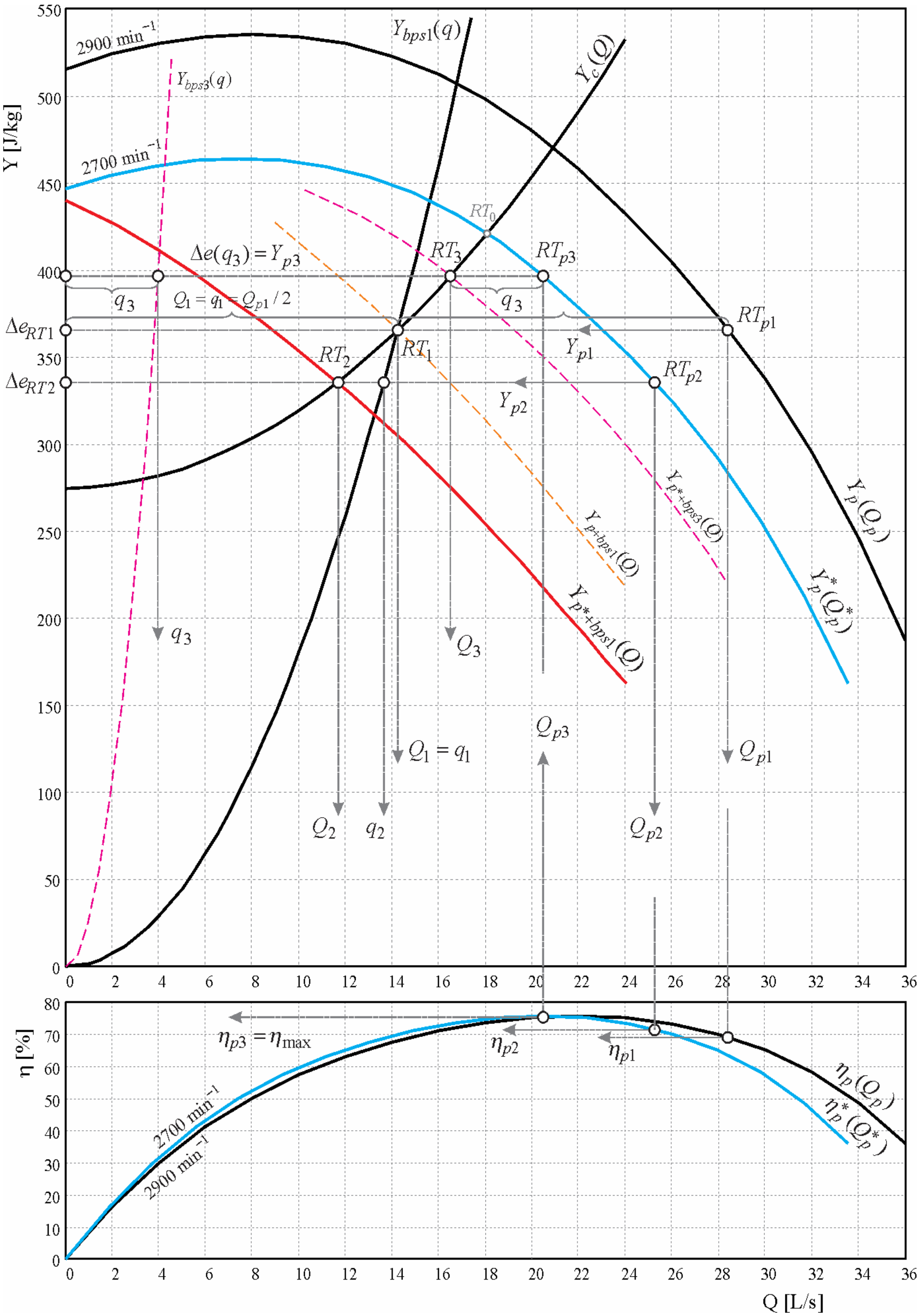
Homework:

1. Solve the problem using continuity equation (6) – parallel connection between system characteristic and bypass characteristic.
2. Find specific energy of pumping E_p [kWh/m^3] in terms when pump is working at n = 2900 rpm and valve coefficient at bypass is ζ = 25. Find specific energy of pumping if valve at main pipeline is suffocated, for the same inflow to reservoir B (bypass is closed)? Compare. In both cases adopt efficiency coefficient of EM that is η_{em} = 91 %.

   Note:

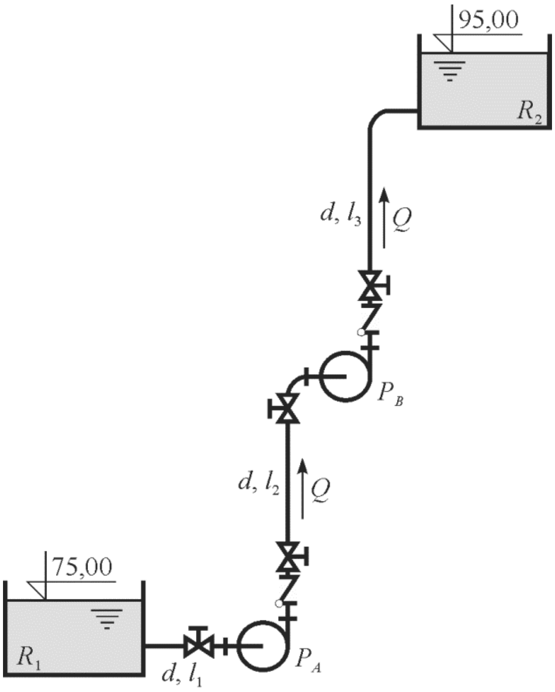
   Specific pumping energy E_p [kWh/m^3] is important indicator of pump system efficiency and represents [kWh] that is needed to pump one m^3 of water to consumer. It can be counted as:

$$E_p = \frac{P_{em}\ [\text{kW}]}{Q\ \left[\text{m}^3/\text{h}\right]} = \frac{P_p}{\eta_{em} Q}$$



Y [J/kg]
2900 min⁻¹
2700 min⁻¹
Ybps1(q)
Ybps3(q)
Yc(Q)
RT0
RT3
RTp3
Δe(q3) = Yp3
q3
Q1 = q1 = Qp1 / 2
ΔeRT1
ΔeRT2
RTp1
RT1
RT2
Yp1
RTp2
Yp2
Yp*+bps3(Q)
Yp+bps1(Q)
Yp*+bps1(Q)
Yp(Qp)
Yp*(Qp*)
q3
Q3
Qp3
Q1 = q1
Qp1
Q2
q2
Qp2
η [%]
ηp3 = ηmax
ηp2
ηp1
ηp(Qp)
ηp*(Qp*)
Q [L/s]

Exercise 5: Two identical centrifugal pumps with known characteristics at $n = 1460$ rpm, are working in serial connection on pipeline shown on Figure. Pipeline consists of three sections with diameter of $d = 300$ mm, total length $l_1 + l_2 + l_3 = 3460$ m and friction resistance coefficient $\lambda = 0{,}023$. Hydraulic local losses are 10% of friction losses.



Find:

1. Flow, heads, and required shaft power of both centrifugal pumps if pump P_B works at $n = 1300$ rpm.
2. Rotational speed of the pump P_B at which inflow to reservoir R_2 is for 15 % greater than in solution (1). What are the shaft powers required then?
3. Rotational speed of the pump P_B at which that pump works with max efficiency. What are the shaft power required then?

Rotational speed of the pump P_A is always $n = 1460$ rpm.

Pump performance characteristics at $n = 1460$ rpm:

Q_p [L/s]	0	50	100	150	200	250
Y_p [J/kg]	393	393	384	364	319	246
η_p [%]	0	35	60	70	67	56

Solution (1):

Problem is solved using Bernoulli equation for points on water surfaces on reservoirs R_1 and R_2.

B.E. *A-K*

$$gz_{R1} + Y_{pA} + Y_{pB} = gz_{R1} + \sum_{R1}^{R2} \Delta y_g(Q)$$

After rearranging so that on the left side are only heads of the pump:

$$Y_{pA} + Y_{pB} = g(z_{R2} - z_{R1}) + (1 + \frac{\varphi}{100}) \frac{8Q^2}{\pi^2 d^4} \lambda \frac{l_1 + l_2 + l_3}{d} = Y_c(Q) \qquad (1)$$

Where are local hydraulic losses estimated as 10% of total friction losses ($\varphi = 10\%$).

Term on the right side of equation (1) represents pipeline characteristic from reservoir R_1 to reservoir $R_2 \rightarrow Y_c(Q)$. Term on the left side represents total specific energy (head) of the pumps at flow Q in this pump system. Connection of the pumps at which heads are being counted at same flows is called serial connection of the pumps.

In thic concrete problem, pump P_A works with rotational speed n = 1460 rpm, with operational characteristics $Y_{pA}(Q_{pA}) = Y_p(Q)$, $\eta_{pA}(Q_{pA}) = \eta(Q)$ given in table, pump P_B works with rotational speed n_{B1} = 1300 min^{-1}, so using affinity laws operational characteristics of the pump can be counted:

$$Q_{pB}^{*} = \frac{n_{B1}}{n} Q_p \qquad Y_{pB}^{*} = \left(\frac{n_{B1}}{n}\right)^2 Y_p \qquad \eta_{pB}^{*} = \eta_p$$

Results are shown in table.

Pump performance characteristics at n = 1460 rpm:

Q_p [L/s]	0	50	100	150	200	250
Y_p [J/kg]	393	393	384	364	319	246
η_p [%]	0	35	60	70	67	56

$\times\frac{1300}{1460}$ (Q), $\times\left(\frac{1300}{1460}\right)^2$ (Y), $=$ (η)

Pump performance characteristics P_B at n_{B1} = 1300 rpm:

Q_{pB}^{*} [L/s]	0	44,5	89,0	133,6	178,1	222,6
Y_{pB}^{*} [J/kg]	311,6	311,6	304,5	288,6	252,9	195,0
η_{pB}^{*} [%]	0	35	60	70	67	56

Operational characteristics of both pumps $Y_{pA}(Q_{pA}) = Y_{pB}(Q_{pB}) = Y_p(Q)$, $\eta_{pA}(Q_{pA}) = \eta_{pB}(Q_{pB}) = \eta(Q)$ at n = 1460 min^{-1}, pump P_B at n_{B1} = 1300 min^{-1} ($Y_{pB}^{*}(Q_{pB}^{*})$, $\eta_{pB}^{*}(Q_{pB}^{*})$) and system characteristic $Y_c(Q)$ are shown on graph.

Following equation (1), pump characteristics $Y_{pA}(Q_{pA})$ and $Y_{pB}^{*}(Q_{pB}^{*})$ are being connected in serial – at same flows, heads are being counted. Result, ie characteristic of this serial connection is shown on graph → $Y_{pA}(Q_{pA}) + Y_{pB}^{*}(Q_{pB}^{*}) = Y_{A+B*}(Q)$.

In intersection of this serial connection of the pumps $Y_{A+B*}(Q)$ and system characteristic $Y_c(Q)$, operational system point is acquired RT_1 , from whom flow can be read Q_1 = 127 L/s. Operaional pump points RT_{pA1} and RT_{pB1} are acquired in intersections of vertical lines at $Q_1 = Q_{pA1} = Q_{pB1} = const$ with appropriate pumo characteristics $Y_{pA}(Q_{pA})$ and $Y_{pB}^{*}(Q_{pB}^{*})$. From operational pump points, flows, heads and efficiency coefficients can be read, and finally required shaft powers can be found:

$$RT_1 \rightarrow RT_{pA1} \rightarrow \qquad Y_{pA1} = 375{,}4\ \frac{\mathrm{J}}{\mathrm{kg}} \qquad Q_{pA1} = Q_1 = 127\frac{\mathrm{L}}{\mathrm{s}} \qquad \eta_{pA1} = 67{,}3\ \%$$

$$P_{pA1} = \frac{\rho Q_{pA1} Y_{pA1}}{\eta_{pA1}} = 70{,}9\ \mathrm{kW}$$

$RT_1 \rightarrow RT_{pB1} \rightarrow \qquad Y_{pB1} = 291{,}9\ \frac{\text{J}}{\text{kg}} \qquad Q_{pB1} = Q_1 = 127\frac{\text{L}}{\text{s}} \qquad \eta_{pB1} = 69{,}5\ \%$

$$P_{pB1} = \frac{\rho Q_{pB1} Y_{pB1}}{\eta_{pB1}} = 53{,}4\ \text{kW}$$

Solution (2):

Operational system point RT_2 is acquired on system characteristic $Y_c(Q)$ from requirement that flow Q_2 is for 15 % greater than flow Q_1:

$$Q_2 = 1{,}15 Q_1 = 1{,}15 \cdot 127\ \text{L/s} = 146{,}1\ \text{L/s} \qquad \rightarrow \qquad RT_2$$

New serial characteristic of pump P_A and pump P_B at unknown rotational speed n_{B2} must go through operational system point $RT_2 \rightarrow Y_{pA}(Q_{pA}) + Y_{pB}^{**}(Q_{pB}^{**}) = Y_{A+B^{**}}(Q)$.

So, total head of these two pumps is:

$$RT_2 \rightarrow \qquad Y_{RT2} = Y_{pA2} + Y_{pB2} = 819{,}3\ \frac{\text{J}}{\text{kg}}$$

Operational pump point P_A can be found in intersection of vertical line at $Q_2 = Q_{pA2} = Q_{pB2} =$ *const* with operational characteristic of this pump $Y_{pA}(Q_{pA})$. From that operational pump point RT_{pA2} head of the pump P_A and efficiency coefficient can be read, so required shaft power can be counted:

$RT_2 \rightarrow RT_{pA2} \rightarrow \qquad Y_{pA2} = 366{,}3\ \frac{\text{J}}{\text{kg}} \qquad Q_{pA2} = Q_2 = 146{,}1\ \frac{\text{L}}{\text{s}} \qquad \eta_{pA2} = 69{,}8\ \%$

$$P_{pA2} = \frac{\rho Q_{pA2} Y_{pA2}}{\eta_{pA2}} = 76{,}7\ \text{kW}$$

Operational pump point P_B can be found from term that pump in this serial connection must have head equal to $Y_{pB2} = Y_{RT2} - Y_{pA2}$ at flow $Q_{pB2} = Q_2$:

$$Y_{pB2} = Y_{RT2} - Y_{pA2} = 453\ \frac{\text{J}}{\text{kg}} \qquad Q_{pB2} = Q_2 = 146{,}1\ \frac{\text{L}}{\text{s}} \quad \rightarrow \quad RT_{pB2}$$

New pump characteristic Y_{pB}^{**} should go through this operational point, at rotatioanl speed n_{B2} (dashed line on graph). To find unknown rotational speed n_{B2}, similar parable must go through operational pump point RT_{pB2}:

$$Y_{s2}(Q_s) = \left(\frac{Y_{pB2}}{Q_{pB2}^2} \right) Q_s^2$$

In intersection of similar parable $Y_{s2}(Q_s)$ and operational characteristic of the pump $Y_{pB}(Q_{pB}) = Y_p(Q_p)$ at n = 1460 rpm, similar point ST_2 is acquired, from whom flow (or head) and efficiency coefficient can be read:

$ST_2 \rightarrow \qquad Q_{s2} = 132{,}6 \ \frac{\text{L}}{\text{s}} \qquad \eta_{s2} = \eta_{pB2} = 68{,}2 \ \%$

Required rotational speed of the pump P_B is acquired from affinity laws:

$$n_{2B} = \frac{Q_{pB2}}{Q_{s2}} n = 1608{,}8 \text{ rpm}$$

Finally required shaft power is:

$$P_{pB2} = \frac{\rho Q_{pB2} Y_{pB2}}{\eta_{pB2}} = 97 \text{ kW}$$

Solution (3):

It is required to find rotational speed of the pump n_{B3} at which pump P_B works in serial connection with pump P_A on given pipeline with max efficiency, while pump P_A works at n = 1460 rpm.

Max efficiency coefficient on efficiency characteristic $\eta_p(Q_p)$ at n = 1460 min^{-1} is:

$$\eta_{pB3} = \eta_{\max} = 70{,}3 \ \%$$

so on operational characteristic $Y_p(Q_p)$ at n = 1460 min^{-1} optimal operational point is found:

$RT_{opt} \rightarrow \qquad Q_{opt} = 160{,}4 \ \frac{\text{L}}{\text{s}} \qquad Y_{opt} = 357{,}0 \ \frac{\text{J}}{\text{kg}}$

If similar parable is put through operational point RT_{opt} :

$$Y_{s3}(Q_s) = \left(\frac{Y_{opt}}{Q_{opt}^2} \right) Q_s^2$$

then, using affinity laws, in all points on that similar parable efficiency coefficient is the same – η_{max}. Another words, required operational pump point P_B must be somewhere on this „optimal“ similar parable. That opens a possibilitiy that similar parable $Y_{s3}(Q_s)$ can be used as imaginary characteristic of pump P_B, so they can be put in serial connection with pump P_A – $Y_{pA}(Q_{pA})$ = $Y_p(Q_p)$.

Characteristic of this serial connection is marked as $[Y_{pA} + Y_{s3}]_R$ on graph. In intersection of this characteristic and system characteristic $Y_c(Q)$ operational system point is found RT_3 , through this point original serial characteristc of pump P_A and pump P_B must pass at unknown rotational speed $n_{B3} \rightarrow Y_{pA}(Q_{pA}) + Y_{pB}^{***}(Q_{pB}^{***}) = Y_{A+B^{***}}(Q)$.

Operational points of pumps P_A and P_B are found in intersection of vertical line at $Q_3 = Q_{pA3} = Q_{pB3} = const$ with pump characteristic $Y_{pA}(Q_{pA}) \rightarrow RT_{pA3}$ and similar parable $Y_{s3}(Q_s) \rightarrow RT_{pB3}$. From operational pump point RT_{pA3} working parameters of the pump P_A can be read and required shaft power can be counted:

$RT_3 \rightarrow RT_{pA3} \rightarrow \qquad Y_{pA3} = 381{,}3 \ \frac{\text{J}}{\text{kg}} \qquad Q_{pA3} = Q_3 = 109{,}9 \ \frac{\text{L}}{\text{s}} \qquad \eta_{pA3} = 63{,}2 \ \%$

$$P_{pA3} = \frac{\rho Q_{pA3} Y_{pA3}}{\eta_{pA3}} = 66{,}3 \text{ kW}$$

From operational pump poin RT_{pB3} (pump characteristic Y_{pB}^{***} is passing through it at unknown rotational speed n_{B3}) flow and head of the pump P_B can be read:

$$RT_3 \rightarrow RT_{pB3} \rightarrow \qquad Y_{pB3} = 167{,}8 \ \frac{\text{J}}{\text{kg}} \qquad Q_{pB3} = Q_3 = 109{,}9 \ \frac{\text{L}}{\text{s}}$$

Efficiency coefficient of the pump P_B is equal to efficiency coefficient of optimal operational point RT_{opt} (operational pump points RT_{pB3} are RT_{opt} found on optimal similar parable Y_{s3}):

$$\eta_{pB3} = \eta_{\max} = 70{,}3 \text{ \%}$$

so required shaft power of the pump P_B is:

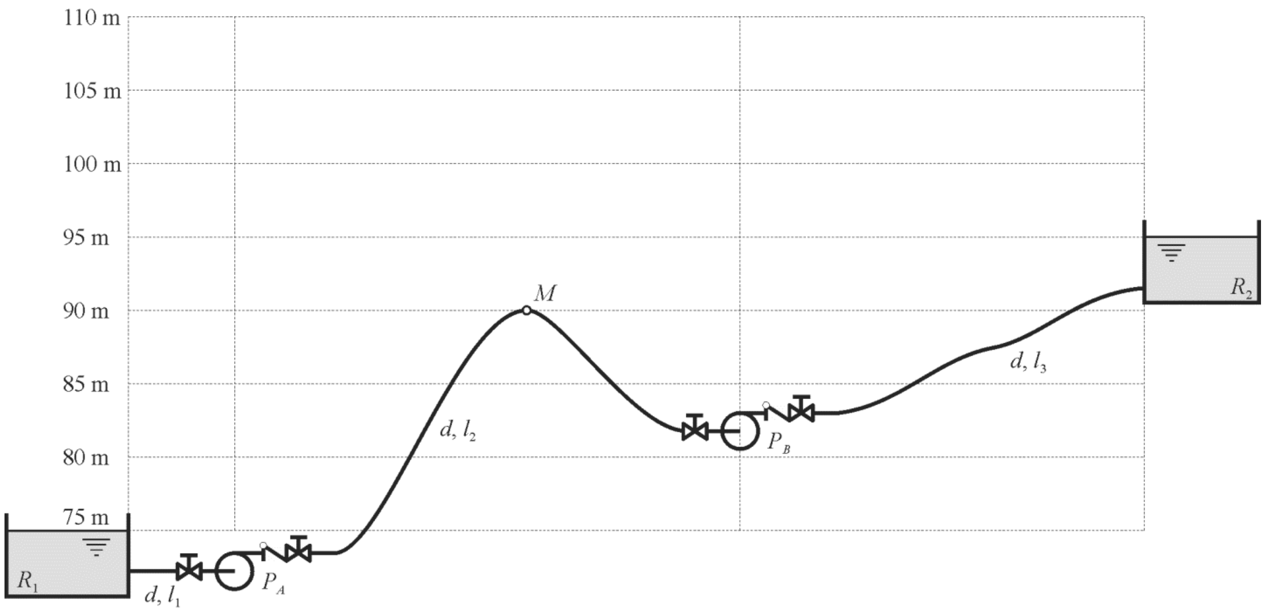
$$P_{pB3} = \frac{\rho Q_{pB3} Y_{pB3}}{\eta_{pB3}} = 26{,}3 \text{ kW}$$

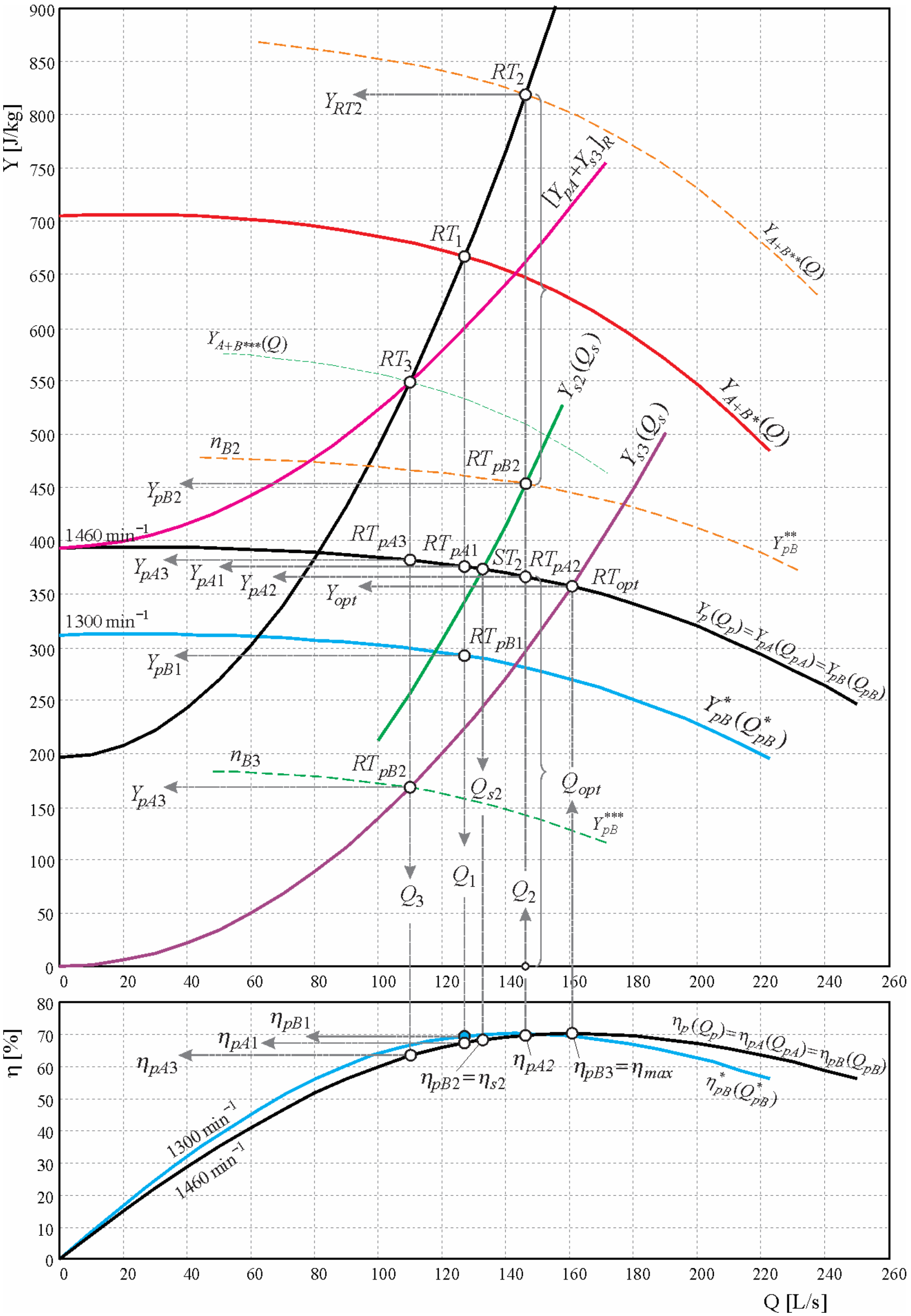
Unknown rotational speed of the pump P_B is found from affinity laws:

$$n_{3B} = \frac{Q_{pB3}}{Q_{opt}} n = 1001 \text{ min}^{-1}$$

Homework:

1. For solution at (1) draw a piezometer line of the system. Lengths of the pipeline sections are l_1= 10 m, l_2 = 1750 m, l_3 = 1700 m. For all sections local hydraulic losses determine at 10% of friction losses. What is the pressure in point M?

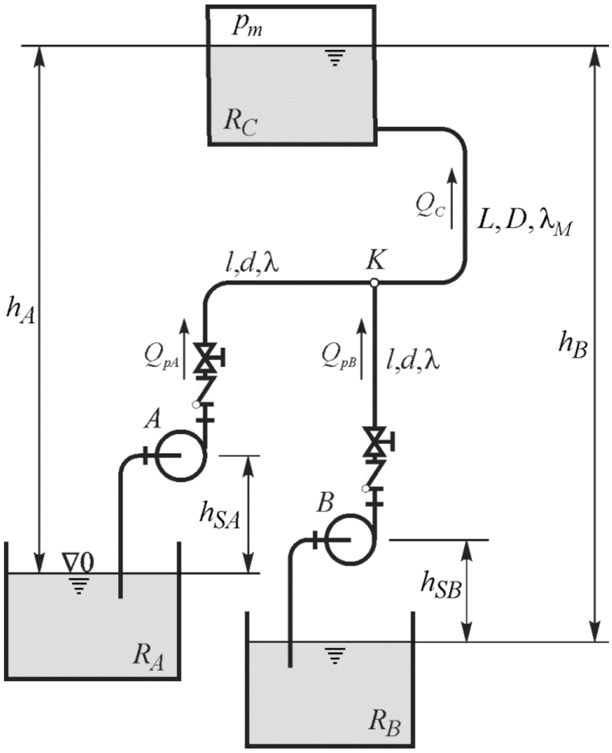




Y [J/kg]
900
850
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
0
20
40
60
80
100
120
140
160
180
200
220
240
260
RT_2
Y_{RT2}
$[Y_{pA}+Y_{s3}]_R$
$Y_{A+B^{**}}(Q)$
RT_1
$Y_{A+B^{***}}(Q)$
RT_3
$Y_{s2}(Q_s)$
$Y_{A+B^{*}}(Q)$
n_{B2}
RT_{pB2}
$Y_{s3}(Q_s)$
Y_{pB2}
1460 min^{-1}
RT_{pA3}
RT_{pA1}
ST_2
RT_{pA2}
Y_{pB}^{**}
Y_{pA3}
Y_{pA1}
Y_{pA2}
Y_{opt}
RT_{opt}
1300 min^{-1}
$Y_p(Q_p)=Y_{pA}(Q_{pA})=Y_{pB}(Q_{pB})$
RT_{pB1}
Y_{pB1}
$Y_{pB}^{*}(Q_{pB}^{*})$
n_{B3}
RT_{pB2}
Y_{pA3}
Q_{s2}
Q_{opt}
Y_{pB}^{***}
Q_3
Q_1
Q_2
η [%]
80
70
60
50
40
30
20
10
0
η_{pB1}
η_{pA1}
η_{pA3}
$\eta_p(Q_p)=\eta_{pA}(Q_{pA})=\eta_{pB}(Q_{pB})$
$\eta_{pB2}=\eta_{s2}$
η_{pA2}
$\eta_{pB3}=\eta_{max}$
$\eta_{pB}^{*}(Q_{pB}^{*})$
1300 min^{-1}
1460 min^{-1}
Q [L/s]

Exercise 6: Two identical centrifugal pumps, with performance characteristics at $n = 960$ rpm, are working in parallel and pumping water from reservoirs R_A and R_B to reservoir R_C. Reservoirs R_A and R_B are open and reservoir R_C is closed at overpressure p_m = 0,2 bar. Water level differences in reservoirs are h_A = 18 m and h_B = 26 m.

Dimensions and characteristics of main pipeline are L = 1100 m, D = 450 mm and λ_M = 0,025. Pump pipelines are identical with next characteristics: l = 100 m, d = 300 mm, λ = 0,03, $\Sigma\zeta$ = 6.



Find:

1. Inflow to reservoir R_C, flows, heads and required shaft powers of pumps if both pumps are working at n = 960 rpm.
2. What must be the rotational speed of the pump A so that the flow through pump B is zero (at n = 960 rpm)?
3. For working regimes at 1, find maximal allowed suction heights of the pumps h_s.

   Lengths of suction pipelines are l_s = 6 m, total local resistances are $\Sigma\zeta_s$ = 4. Saturated steam pressure is p_{zp} = 0,024 bar, and atmospheric pressure is p_a = 990 mbar. Safety addon is S = 1 m.

Pump performance characteristics at n = 960 rpm

Q [L/s]	0	40	80	120	140	160	180	200	220
Y [J/kg]	392	422	422	392	363	324	275	216	147
η [%]	0	47	70	80	81	80	75	65	50
NPSHR [m]		2,5	1,8	2,3	2,8	3,6	4,7	5,9	7,3

Solution (1):

Problem is solved using Bernoulli equation and Continuity equation. Bernoulli equations are written for all sections of pipelines separately, so that every equation has only one unknown flow figuring.

This problem has two reservoirs in suction part of the pump so it is necessary to chose reservoir from whose surface shall we measure heights. In this case it will be reservoir R_A.

Taking in consideration relative pressures instead of absolute pressures, Bernoulli equation from water level in reservoir R_A to bifurcation point K is:

$$Y_{pA} = e_K(Q_{pA}) + \frac{8Q_{pA}^2}{\pi^2 d^4}\left(\lambda \frac{l}{d} + \Sigma\zeta\right)$$

Equation above should be rearranged so that on the left side is only found specific energy in bifurcation point $e_K(Q_{pA})$:

$$e_{Km}(Q_{pA}) = Y_{pA} - \frac{8Q_{pA}^2}{\pi^2 d^4}\left(\lambda \frac{l}{d} + \Sigma\zeta\right) = Y_{pA} - \Delta y_{gA}(Q_{pA}) = Y_{prA}(Q_{pA}) \tag{1}$$

Function on the right side is reduced pump characteristic Y_{prA} (pump performance characteristic reduced for hydraulic losses in pump pipeline):

Bernoulli equation from water level in reservoir R_B to bifurcation point K:

$$-g(h_B - h_A) + Y_{pB} = e_K(Q_{pB}) + \frac{8Q_{pB}^2}{\pi^2 d^4}\left(\lambda \frac{l}{d} + \Sigma\zeta\right)$$

By rearranging equation $e_K(Q_{pB})$, on the right side is function that conditionally can be named reduced pump characteristic Y_{prB} . In this case that is pump characteristic reduced for hydraulic losses and for difference in height between water level in reservoir R_B and ground level:

$$\begin{aligned} e_K(Q_{pB}) &= Y_{pB} - g(h_B - h_A) - \frac{8Q_{pB}^2}{\pi^2 d^4}\left(\lambda \frac{l}{d} + \Sigma\zeta\right) = \\ &= Y_{pB} - g(h_B - h_A) - \Delta y_{gB}(Q_{pB}) = Y_{prB}(Q_{pB}) \end{aligned} \tag{2}$$

Finally Bernoulli equation from bifurcation point K to water level in reservoir R_C is:

$$e_K(Q_C) = \frac{p_m}{\rho} + gh_A + \frac{8Q_C^2}{\pi^2 D^4}\lambda_M \frac{L}{D} = Y_{cm}(Q_C) \tag{3}$$

Function on the right side $Y_{cm}(Q_C)$ represents system characteristic of the main pipeline K-R_C.

Continuity equation for bifurcation point K is:

$$Q_{pA} + Q_{pB} = Q_C \tag{4}$$

Pump characteristics $Y_p(Q_p)$, $\eta_p(Q_p)$ and $NPSHR(Q_p)$ (pumps A and B are identical), reduced pump characteristics $Y_{prA}(Q_{pA})$ and $Y_{prB}(Q_{pB})$, loss characteristics in pump pipelines $\Delta y_{gA}(Q_{pA}) = \Delta y_{gB}(Q_{pB})$ and system characteristic of main pipeline $Y_{cm}(Q_C)$ are shown graphically.

In stable regime of this pump system left sides of equations (1), (2) and (3) numerically must be the same (specific energy in bifurcation point K) so appropriate characteristics on the right sides of functions must also be the same. Another words, reduced characteristics of the pumps and system characteristic can be put in parallel. Following the rule, reduced pump characteristics are put in parallel $Y_{prA}(Q_{pA})$ and $Y_{prB}(Q_{pB})$ following continuity equation (4), at same head (e_K) flows are counted Q_{pA} and Q_{pB}. Result, parallel characteristic of reduced pump characteristics is shown graphically → $Y_{prAB}(Q_{pA} + Q_{pB}) = Y_{prAB}(Q_C)$.

In intersection of parallel characteristic of the pumps $Y_{cAB}(Q_p)$ and system characteristic of the main pipeline $Y_{cm}(Q_C)$ operational system point RT_1 is acquired.

Conditionally it can be said that operational point RT_1 is found in intersection of imagined pump characteristic (which is sum of both pump performance characteristics and their pipelines) and main pipeline (system) characteristic. In that way problem is reduced to system of one pump working in simple pipeline (exercise 1).

From operational system point RT_1 flow to reservoir R_C can be read:

$$RT_1 \rightarrow \qquad Q_{C1} = 277{,}9\,\frac{\mathrm{L}}{\mathrm{s}}$$

Operational points of pumps RT_{pA1} and RT_{pB1} are obtained on pump performance characteristics $Y_{pA}(Q_{pA}) = Y_{pB}(Q_{pB})$ by drawing horizontal line from operational system point RT_1 , $e_K = e_{RT1} = const$ to intersection with reduced pump characteristics $Y_{prA}(Q_{pA})$ и $Y_{prB}(Q_{pB})$, and then from acquired intersection, vertical lines are drawn ($Q_{pA} = const$, $Q_{pB} = const$) to common pump performance characteristic (pumps *A* and *B* are identical).

From operational pump points RT_{pA1} и RT_{pB1} flows, heads and efficiency coefficients can be read from whom required shaft powers can be calculated:

$$RT_1 \rightarrow RT_{pA1} \rightarrow \qquad Y_{pA1} = 329{,}5\,\frac{\mathrm{J}}{\mathrm{kg}} \qquad Q_{pA1} = 157{,}5\,\frac{\mathrm{L}}{\mathrm{s}} \qquad \eta_{pA1} = 80{,}3\ \%$$

$$P_{pA1} = \frac{\rho Q_{pA1} Y_{pA1}}{\eta_{pA1}} = 64{,}6\ \mathrm{kW}$$

$$RT_1 \rightarrow RT_{pB1} \rightarrow \qquad Y_{pB1} = 391{,}5\,\frac{\mathrm{J}}{\mathrm{kg}} \qquad Q_{pB1} = 120{,}4\,\frac{\mathrm{L}}{\mathrm{s}} \qquad \eta_{pB1} = 80{,}0\ \%$$

$$P_{pB1} = \frac{\rho Q_{pB1} Y_{pB1}}{\eta_{pB1}} = 58{,}9\ \mathrm{kW}$$

Solution (2):

From term that $Q_{pB2} = 0$, ie $Q_{C2} = Q_{pA2}$ operational system point can be acquired RT_2, shown on graph (in intersection of horizontal line drawn from $Q_{pB2} = 0$ on reduced pump characteristic $Y_{prB}(Q_{pB})$ and main pipeline (system) characteristic). Using equation (1), operational pump point RT_{pA2}, at unknown rotational speed n_{A2}, must be above operational system point for losses in pump pipeline Δy_{gA} at flow $Q_{pA2} = Q_{C2}$. From that operational point RT_{pA2} required working parameters of the pump *A* can be read:

$$RT_{pA2} \rightarrow \qquad Y_{pA2} = 468{,}6\,\frac{\mathrm{J}}{\mathrm{kg}} \qquad Q_{pA2} = 311{,}3\,\frac{\mathrm{L}}{\mathrm{s}}$$

To find unknown rotational speed of the pump, “similar parable” must go through operational pump point RT_{pA2} (see graph):

$$Y_s(Q_S) = \left(\frac{Y_{pA2}}{Q_{pA2}^2}\right) Q_s^2$$

In intersection of similar parable $Y_s(Q_s)$ and pump performance characteristic $Y_{pA}(Q_{pA})$ at $n_A = 960\ \mathrm{min}^{-1}$, similar point ST_2 is acquired, from which flow is read (or head):

$$ST_2 \rightarrow \qquad Q_{s2} = 204{,}3 \frac{\text{L}}{\text{s}}$$

Required rotational speed of the pump is obtained from affinity laws:

$$n_{A2} = \frac{Q_{pA2}}{Q_{s2}} n_A = 1462{,}4 \text{ min}^{-1}$$

Solution (3):

In general case non-cavitation working pump condition is:

$$NPSHA(Q_p) > NPSHR(Q_p) + S \tag{5}$$

where: $NPSHA(Q_p)$ – Net Positive Suction Head Available,
$NPSHR(Q_p)$ – Net Positive Suction Head Required and
S – safety add-on, depends on type and material of the pump impeller.

Available cavity reserve of system represents overall positive specific energy of fluid at entrance in the pump (y J/N = m:

$$NPSHA(Q_p) = \frac{p_a + p_{mR} - p_{zp}}{\rho g} + \frac{v_R^2}{2} - h_s - \Delta h_{gs}(Q_p) \tag{6}$$

where: p_{mR} – overpressure in reservoir of the pump (usually p_{mR} = 0 Pa),
p_{zp} – saturated steam pressure of fluid at given temperature,
v_R – fluid velocity in reservoir (usually $v_R \approx 0$ m/s),
h_s – suction height of the pump (vertical distance from water level in reservoir to pump) and
$\Delta h_{gs}(Q_p)$ – hydraulic losses in suction pipeline of the pump at flow Q_p.

By replacing the term (6) in equation (5) and solving it by unknown suction height h_s, after neglecting terms p_{mR} and v_R , next equation is acquired:

$$h_s < \frac{p_a - p_{zp}}{\rho g} - \Delta h_{gs}(Q_p) - NPSHR(Q_p) - S \tag{7}$$

In this problem, required cavity reserves of the pump are read from the cavity characteristic on the graph for known flows acquired in solution (1):

$$RT_{pA1} \rightarrow \qquad Q_{pA1} = 157{,}5 \frac{\text{L}}{\text{s}} \qquad NPSHR_{pA1} = 3{,}5 \text{ m}$$

$$RT_{pB1} \rightarrow \qquad Q_{pB1} = 120{,}4 \frac{\text{L}}{\text{s}} \qquad NPSHR_{pB1} = 2{,}3 \text{ m}$$

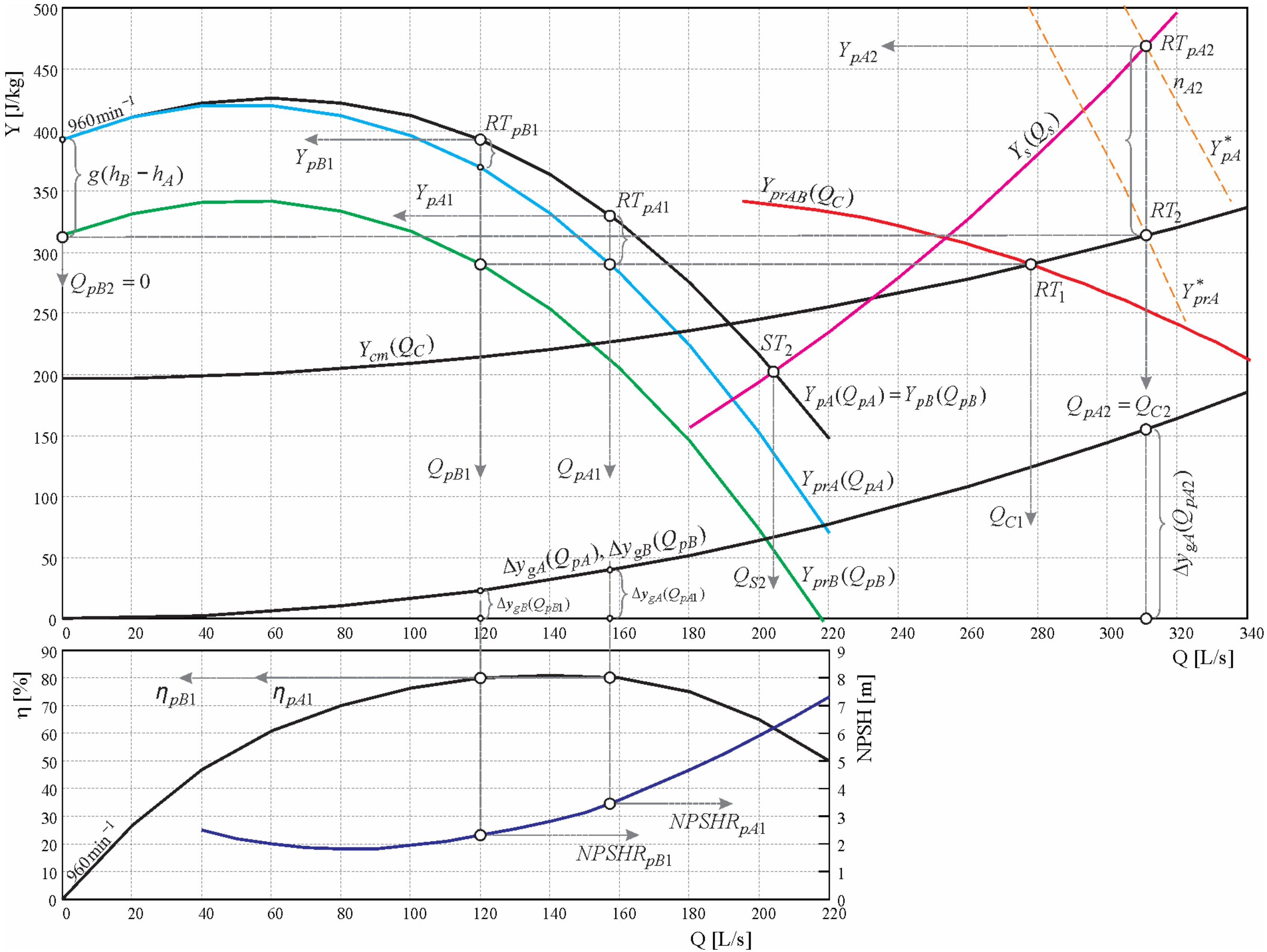
By replacing flows and required cavity reserves in equation (7) maximal suction heights are calculated:

$$h_{s.pA1} < \frac{p_a - p_{zp}}{\rho g} - \frac{8Q_{pA1}^2}{g\pi^2 d^4}\left(\lambda \frac{l_s}{d} + \sum \zeta_s\right) - NPSHR_{pA1}(Q_{pA1}) - S = 4{,}2 \text{ m}$$

$$h_{s.pB1} < \frac{p_a - p_{zp}}{\rho g} - \frac{8Q_{pB1}^2}{g\pi^2 d^4}\left(\lambda \frac{l_s}{d} + \sum \zeta_s\right) - NPSHR_{pB1}(Q_{pB1}) - S = 5{,}9 \text{ m}$$

Homework:

1. Problems at (1) and (2) solve by setting the ground level on water surface in reservoir R_B.
2. Find required shaft power for operational pump point RT_{pA2}.

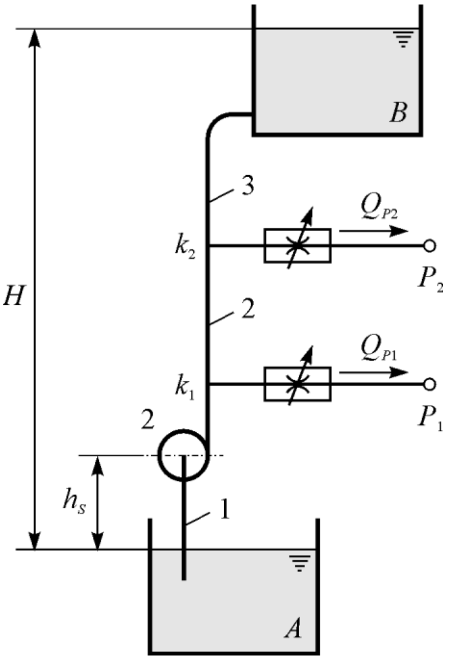

Y [J/kg]
960min^{-1}
$g(h_B - h_A)$
$Q_{pB2} = 0$
RT_{pB1}
Y_{pB1}
Y_{pA1}
RT_{pA1}
$Y_{cm}(Q_C)$
Q_{pB1}
Q_{pA1}
$\Delta y_{gA}(Q_{pA}), \Delta y_{gB}(Q_{pB})$
$\Delta y_{gB}(Q_{pB1})$
$\Delta y_{gA}(Q_{pA1})$
$Y_{prAB}(Q_C)$
ST_2
$Y_{pA}(Q_{pA}) = Y_{pB}(Q_{pB})$
$Y_{prA}(Q_{pA})$
$Y_{prB}(Q_{pB})$
Q_{S2}
$Y_s(Q_s)$
RT_1
Q_{C1}
Y_{pA2}
RT_{pA2}
n_{A2}
Y^*_{pA}
RT_2
Y^*_{prA}
$Q_{pA2} = Q_{C2}$
$\Delta y_{gA}(Q_{pA2})$
Q [L/s]
η [%]
NPSH [m]
η_{pB1}
η_{pA1}
960min^{-1}
$NPSHR_{pA1}$
$NPSHR_{pB1}$
Q [L/s]

Exercise 7: Centrifugal pump whose head, power and net positive suction head characteristics, at n=2900 rpm, are given in next equations (flow is given in L/s):

$Y = 1056 - 0{,}237\ Q^2$ [J/kg]

$P = 10 + 2{,}303\ Q^{0{,}714}$ [kW]

$NPSH_R = 2{,}2 + 0{,}005\ Q^{1{,}8}$ [m]



works in installation which supports users P_1 and P_2 with water. Resistance characteristics of pipelines are given:

$$b_1 = 0{,}1\frac{J/kg}{(L/s)^2};\ b_2 = 0{,}09\frac{J/kg}{(L/s)^2};\ b_3 = 0{,}241\frac{J/kg}{(L/s)^2}.$$

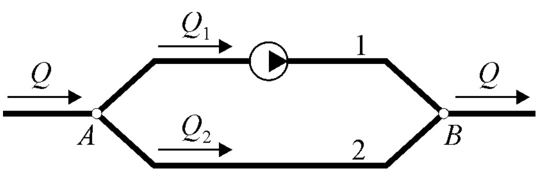
Elevation of reservoir B is at H = 60 m.

Problems:

a) Find flow, head and efficiency of the pump, if inflows to users P_1 и P_2 are constant and are Q_{P1} = 5 L/s, Q_{P2} = 7 L/s;

b) Find the allowable suction height relative to water level in reservoir A (atmospheric pressure is p_a = 1 bar, vapour pressure is p_{zp} = 0,02 bar, loss coefficient in suction part of the pump is $b_1/2$).

Exercise 8: Nodes *A* и *B* of the pipeline are connected with two identical pipes 1 и 2, l = 20 m, d = 50 mm, λ = 0,03. Centrifugal pump with known characteristic performances at n = 800 rpm is connected to pipe 1. Find:



1. Flow in pipes 1 и 2 and head of the pump if total inflow in node A is Q = 12 L/s, and in case if that inflow is Q = 3 L/s.

2. At which rotation speed of the pump total flow Q = 12 L/s goes through pump (pipe 1), and through pipe 2 null.

Pump performance characteristics n = 800 min^{-1}:

Q [L/s]	0	2	4	6	8	10
Y [J/kg]	75	79	78	69	45	14